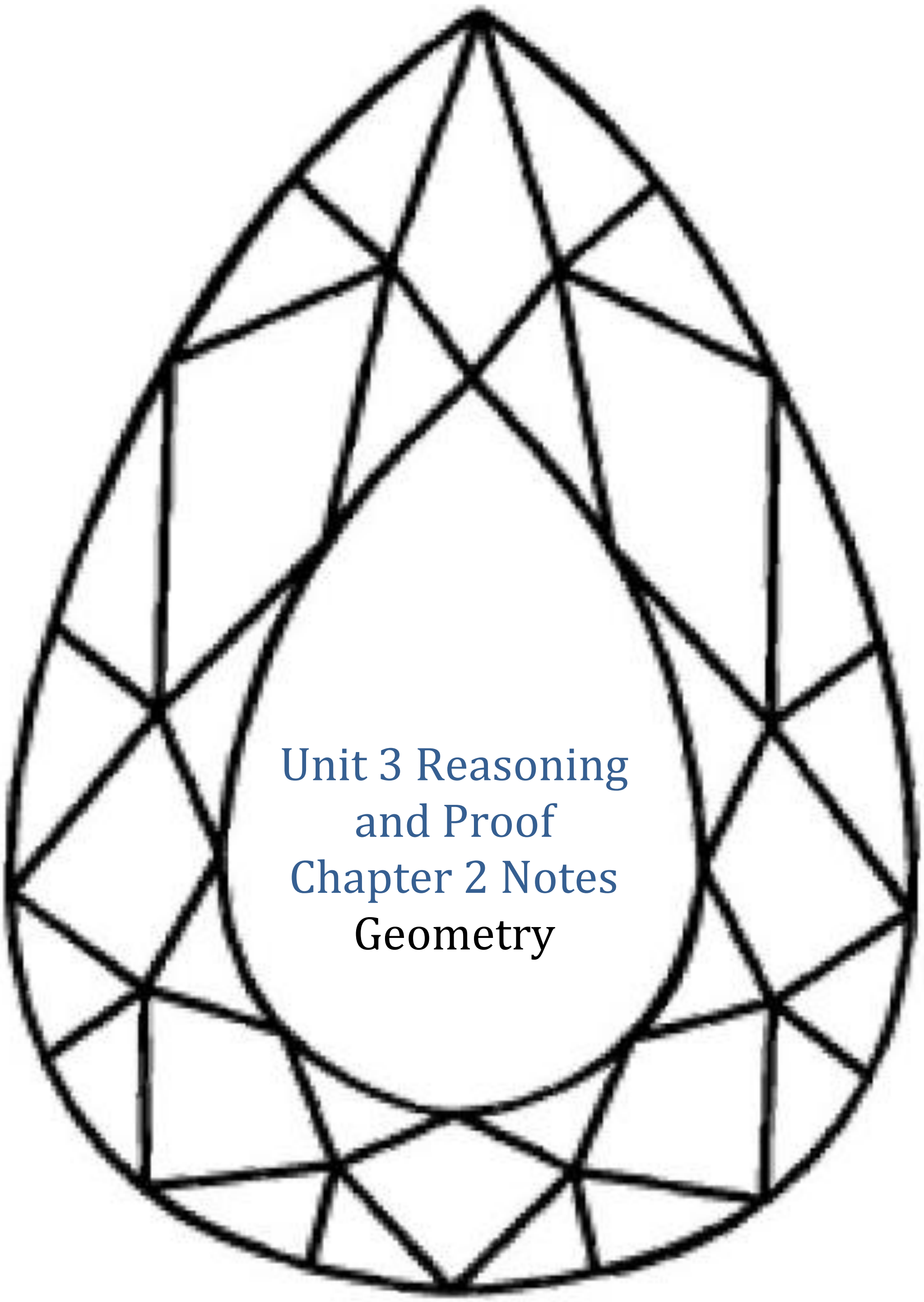


Unit 3 Reasoning and Proof Chapter 2 Notes

Geometry

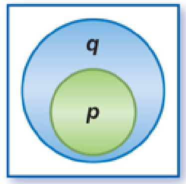
Name ______________________________ Hour __________

Monday **September 30**	2-3 Conditional Statements
Tuesday **October 1**	2-5 Postulates and Proof DHQ 2-3
Block **Wed/Thurs.** **Oct 2/3**	2-6 Algebraic Proof DHQ 2-5 UNO Proof Activity
Friday **October 4**	Quiz 2-3 to 2-6
Monday **October 7**	2-7 Segment Proof
Tuesday **October 8**	2-8 Angle Proof DHQ 2-7
Block **Wed/Thurs.** **October 9/10**	<u>Proof Practice</u> DHQ 2-8
Friday **October 11**	No School – Teacher Work Day
Monday **October 14**	Proof Review
Tuesday **October 15**	ACT Test Tips
Wednesday **October 16**	ASVAB/PSAT/ACT Practice Parent Teacher Conferences 4:30-8:30
Thursday/Friday **October 17-18**	No School Thursday, Parent Teacher Conferences 11-8
Monday **October 21**	Proof Review for Test
Tuesday **October 22**	Proof Review for Test
Wed/Thurs **October 23/24**	Proof Test

2.3 Conditional Statements

<u>I can</u> analyze statements in if-then form.

<u>I can</u> write the converse of if-then statements



$p \rightarrow q$

Conditional Statement- an if-then statement ($p \rightarrow q$)

Hypothesis – is the phrase immediately following the word "if"

Conclusion – is the phrase immediately following the word "then"

<u>Example 2-3-1</u> Circle the hypothesis and underline the conclusion in the following sentences.

a. If a polygon has 6 sides, then it is a hexagon.

b. Tamika will advance to the next level of play if she completes the maze in her computer game.

c. If today is Thanksgiving Day, then today is Thursday.

d. A number is a rational number if it is an integer.

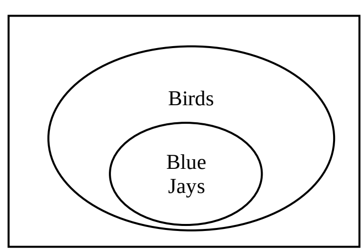
<u>Example 2-3-2</u> Write each statement in if-then form.

a) A five-sided polygon is a pentagon.

b) An angle that measures 45° is an acute angle.

c) An obtuse triangle has exactly one obtuse angle.

d)



ALERT: A conditional with a ***<u>false</u>*** hypothesis is always ***<u>true</u>***.

	<u>Example 2-3-3</u> Determine the truth value of the conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

If last month was February, then this month is March. True/False	When a rectangle has an obtuse angle, it is a parallelogram. True/False
If Mrs. McWhorter teaches geometry, then everyone has a "C" in her class. True/False	If 2+2=7, then a banana is a vegetable. True/False
If two angles are acute, then they are congruent. True/False	If an even number greater than 2 is prime, then 5 + 4 = 8. True/False

Converse of a Conditional Statement.

Converse – The statement formed by ______________________ the _______________ and the _________________ of a conditional statement.

<u>Example 2-3-4</u> Conditional Statements

Write the conditional and its converse for the following true statement. Determine the truth value of each statement. If a statement is false, give a counterexample.

a. Bats are animals that can fly.

Statement (T / F) **Converse (T / F)**

b. The deer population increases with the food available.

Statement (T / F) **Converse (T / F)**

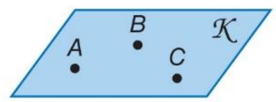
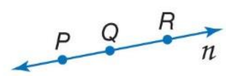
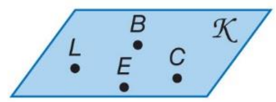
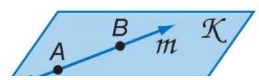
All __definitions__ are **Biconditional Statements.** Statements whose converse is also true, can be written as a biconditional.	**Statements that use the phrase "if and only if" which is abbreviated "iff"** Def of ≅ segments: Segments are ________ **if and only if** segments have the same measure. Def of ≅ angles: Angles are ________ **if and only if** angles have the same measure. Def of right angle: an angle is a _________ angle **iff** the angle's measure is 90°. Def of segment bisector: a segment/ray bisects a segment **iff** the segment/ray goes through the ___________ Def of ∠ bisector: a segment/ray bisects an ∠ **iff** the segment/ray cuts the ______________ perfectly in __________. Def of Complementary: two angles are complementary **iff** the sum of the angles is __________________. Def of Supplementary: two angles are supplementary **iff** the sum of the angles is __________________.

2.5 Postulates and Paragraph Proofs

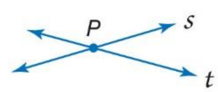
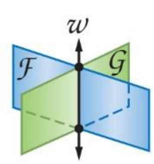
I can identify and use basic postulates about points, lines, and planes.

I can write paragraph proofs.

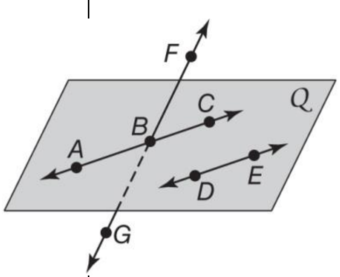
Postulates Points, Lines, and Planes

Words	Example	
2.1 Through any two points, there is exactly one line.		Line n is the only line through points P and R.
2.2 Through any three noncollinear points, there is exactly one plane.		Plane $\mathcal{K}$ is the only plane through noncollinear points A, B, and C.
2.3 A line contains at least two points.		Line n contains points P, Q, and R.
2.4 A plane contains at least three noncollinear points.		Plane $\mathcal{K}$ contains noncollinear points L, B, C, and E.
2.5 If two points lie in a plane, then the entire line containing those		Points A and B lie in plane $\mathcal{K}$, and line m contains points A

KeyConcept Intersections of Lines and Planes

Words	Example	
2.6 If two lines intersect, then their intersection is exactly one point.		Lines s and t intersect at point P.
2.7 If two planes intersect, then their intersection is a line.		Planes $\mathcal{F}$ and $\mathcal{G}$ intersect in line w.

Example 2-5-1 Identifying Postulates



In the figure, $\overleftrightarrow{AC}$ and $\overleftrightarrow{DE}$ are in plane Q and $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$. State whether each postulate is true or false, then state the postulate that can be used to show each statement is true or false.

a. Exactly one plane contains points F, B, and E.

b. $\overleftrightarrow{BE}$ lies in plane Q.

c. Through points F, B, and G there is exactly one plane

d. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ intersect at B

Example 2-5-2 Analyze Statements Using Postulates
Determine whether the following statement is *always, sometimes,* or *never* true. Explain.

A. If plane T contains $\overleftrightarrow{EF}$ and $\overleftrightarrow{EF}$ contains point G, then plane T contains point G.

B. $\overleftrightarrow{GH}$ contains three noncollinear points.

C. There is exactly one plane that contains points A, B, and C.

D. Points E and F are contained in exactly one line.

E. Two lines intersect in two distinct points M and N.

F. The intersection of plane M and plane N is point A.

G. If A and B lie in plane W, then $\overleftrightarrow{AB}$ lies in plane W.

H. $\overleftrightarrow{TR}$ lies in plane M.

KeyConcept The Proof Process

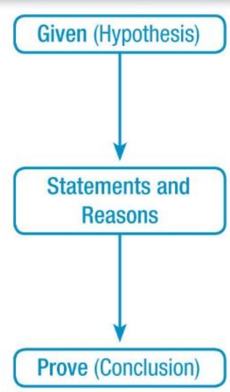
Step 1 List the given information and, if possible, draw a diagram to illustrate this information.

Step 2 State the theorem or conjecture to be proven.

Step 3 Create a deductive argument by forming a logical chain of statements linking the given to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.

Step 5 State what it is that you have proven.



<u>Example 2-5-3</u> Write a Paragraph Proof

A. Given $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{CD}$, write a paragraph proof to show that A, C, and D determine a plane.

   It is given $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{CD}$, so they must intersect at point C, by Postulate ________. So Point A is on $\overleftrightarrow{AC}$ and point D is on $\overleftrightarrow{CD}$. Points A, C, and D are _____ ________________. Therefore,

B. Given that M is the midpoint of $\overline{XY}$, write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

   If M is the midpoint of $\overline{XY}$, then from the definition of midpoint of a segment, we know that _______= ______, This means that _____ and ______have the same measure. By the definition of congruent segments, we know that if the segments have the same measure they are congruent. Therefore, _______≅ ______.

Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs, the conjecture in example 3 is known as the midpoint theorem.

Theorem 2.1 Midpoint Theorem

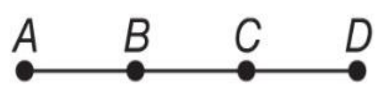
If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Recall:

Transitive property is needed for this proof…keep or not?

<u>Example 2-5-4</u> Apply the Midpoint Theorem

In the figure below, point *B* is the midpoint of $\overleftrightarrow{AC}$ and point *C* is the midpoint of $\overleftrightarrow{BD}$. Write a paragraph proof to prove that *AB* = *CD*.



It is given that point *B* is the midpoint of $\overleftrightarrow{AC}$ and point *C* is the midpoint of $\overleftrightarrow{BD}$. So, $AB = BC$ and $BC = BD$.

	2.6 Algebraic Proof
<u>I can</u> use algebra to write two column proofs. **<u>I can</u> use properties of equality to write geometric proofs**	

Properties of Real Numbers
the following properties are true for any real numbers a, b, and c

Addition Property of Equality	
Subtraction Property of Equality	
Multiplication Property of Equality	
Division Property of Equality	
Reflexive Property of Equality	
Symmetric Property of Equality	
Transitive Property of Equality	
Substitution Property of Equality	
Distributive	

Example 2-6-1 Algebraic Proof
Justify each step using a two column proof.

Given: $2(5 - 3a) - 4(a + 7) = 92$
Prove: $a = -11$

Statements	Reasons

Example 2-6-2 Literal Equations

If the distance d an object travels is given by $d = 20t + 5$, the time t that the object travels is given by $t = \frac{d-5}{20}$. Write a two column proof to verify this conjecture.

Begin by stating what is given and what you are to prove.

Statements	**Reasons**
1. $d = 20t + 5$	1. Given
2.	2.
3.	3.
4. $t = \frac{d-5}{20}$	4.

If the formula for area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, then what would be the formula in terms of height (h). Fill in the missing statements and reasons in the two column proof.

Statements	**Reasons**
1.	1.
2.	2. Multiplication Property of Equality
3. $\frac{2A}{(b_1+b_2)} = h$	3.
4.	4. Symmetric Property of Equality

Example 2-6-3 Geometric Proof

Use a two column proof to verify each conjecture.

Given: $\angle A \cong \angle B, m\angle B = 2m\angle C, m\angle C = 45$

Prove: $m\angle A = 90$

Statements	Reasons
1. $\angle A \cong \angle B$, $m\angle B = 2m\angle C$, $m\angle C = 45$	1.
2.	2. Def of ≅ angles
3. $m\angle A = 2m\angle C$	3.
4.	4. Substitution
5. $m\angle A = 90$	**5.**

Given: $\overline{AB} \cong \overline{CD}$, $\overline{CD} \cong \overline{RS}$, $AB = 12$

Prove: RS = 12

Statements	Reasons
1.	1.
2.	2. Def of ≅ segments
3. $AB = RS$	3.
4.	4. Substitution
5. $RS = 12$	**5.**

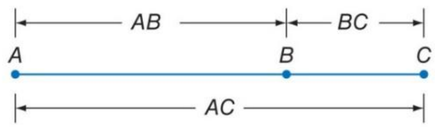
I can write proofs involving segment addition and segment congruence

In Chapter 1 we learned about the Segment Addition Postulate. It is used as a justification in many proofs.

Postulate 2.9 Segment Addition Postulate

Words If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

Symbols



In section 2.6, these same properties were introduced as properties of equality, now they can be used as properties of congruence.

Theorem 2.2 Properties of Segment Congruence

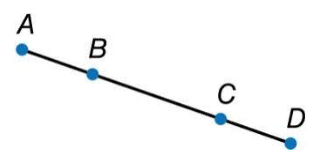
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 2-7-1 Using the Segment Addition Postulate

PROVE that if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

Given: $\overline{AB} \cong \overline{CD}$

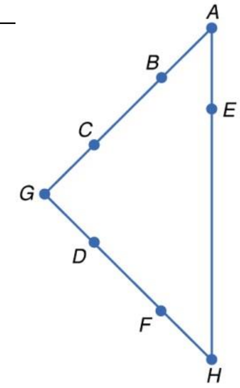
Prove: $\overline{AC} \cong \overline{BD}$



Statements	Reasons

Example 2-7-2: Prove the following
Given: $\overline{GD} \cong \overline{BC}$
$\overline{BC} \cong \overline{FH}$
$\overline{FH} \cong \overline{AE}$

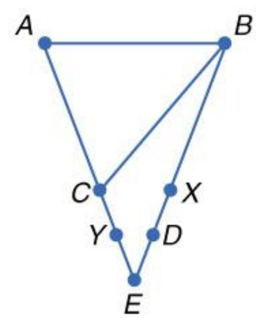
Prove: $\overline{AE} \cong \overline{GD}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	**5.**

Given: $AC = AB$
$AB = BX$
$CY = XD$

Prove: $AY = BD$



Write a paragraph proof to solve the argument.

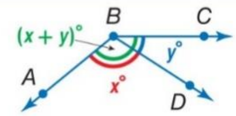
2.8 Proving Angle Relationships

I can write proofs involving supplementary and complementary angles

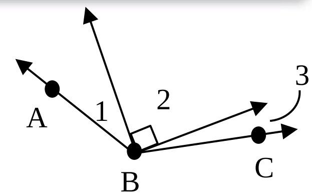
I can write proofs involving congruent and right angles.

Postulate 2.11 Angle Addition Postulate

D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.

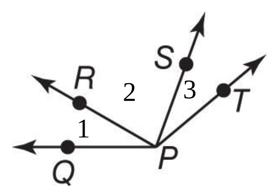


Example 2-8-1: Using the Angle Addition Postulate
Given: $m\angle 1 = 23°$, $\angle 2$ is a right angle, $m\angle ABC = 131°$
Prove: $m\angle 3 = 18°$



Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Example 2-8-2
Given: $\angle QPS \cong \angle TPR$
Prove: $\angle 1 \cong \angle 3$



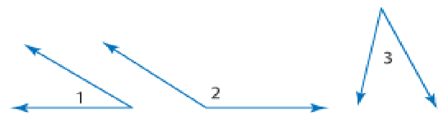
Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Definition of Complementary Angles- Two angles are complementary if and only if the sum of their

Definition of Supplementary Angles- Two angles are supplementary if and only if the sum of their

DEFINITIONS WORK FORWARDS AND BACKWARDS – ARE BICONDITIONAL

Example 2-8-3:
Given: ∠1 and ∠2 are supplementary.
∠2 and ∠3 are supplementary.
Prove: ∠1 ≅ ∠3



Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Theorems

2.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.

2.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.

Congruent Supplements Theorem:
If ∠5 and ∠6 are supplementary and ∠7 and ∠6 are supplementary

then →

______________________________ **:**

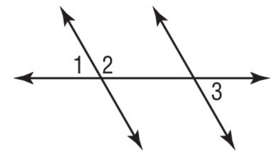
If ∠1 and ∠4 are complementary and ∠1 and ∠2 are complementary

then →

Linear Pair Theorem:
If two angles form a linear pair, then they are supplementary.

∠1 and ∠2 form a ______________________________,

Example 2-8-4
Given: ∠1 and ∠2 form a linear pair.
∠2 and ∠3 are supplementary.
Prove: ∠1 ≅ ∠3



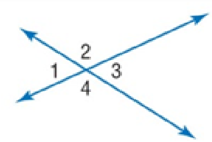
Statements	**Reasons**
a. ∠1 and ∠2 form a linear pair	**a.**
b.	**b.** linear pair theorem
c. ∠2 and ∠3 are supplementary	**c.** given
d.	**d.**

Theorem 2.8 Vertical Angles Theorem

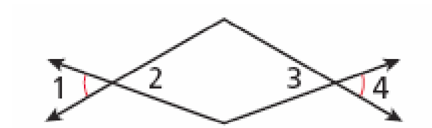
If two angles are vertical angles, then they are congruent.

Abbreviation *Vert. ∡ are ≅.*

Example ∠1 ≅ ∠3 and ∠2 ≅ ∠4

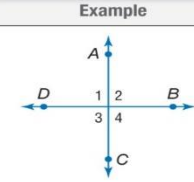
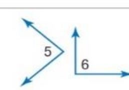
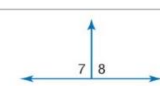


Example 2-8-5 Given: ∠1 ≅ ∠4
Prove: ∠2 ≅ ∠3

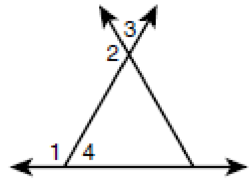


Statements	Reasons
1.	1.
2.	2.
3.	3.

Theorems Right Angle Theorems

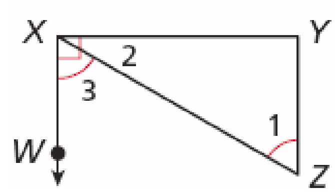
Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then ∠1, ∠2, ∠3, and ∠4 are rt. ∡.	
2.10 All right angles are congruent. **Example** If ∠1, ∠2, ∠3, and ∠4 are rt. ∡, then ∠1 ≅ ∠2 ≅ ∠3 ≅ ∠4.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then ∠1 ≅ ∠2, ∠2 ≅ ∠4, ∠3 ≅ ∠4, and ∠1 ≅ ∠3.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If ∠5 ≅ ∠6 and ∠5 is suppl. to ∠6, then ∠5 and ∠6 are rt. ∡.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If ∠7 and ∠8 form a linear pair, then ∠7 and ∠8 are rt. ∡.	

Example 2-8-6:
Given: ∠4 ≅ ∠3
Prove: ∠1 ≅ ∠2



Statements	Reasons
1.	1.
2.	2. Definition of congruent angles
3. ∠1 and ∠ ____ are supplementary ∠2 and ∠ ____ are supplementary	3.
4.	4. Definition of supplementary
5.	5.
6.	6.
7.	7.
8.	8.

Given: ∠WXY is a right angle. ∠**1** ≅ ∠**3**
Prove: ∠1 and ∠2 are complementary



Statements	**Reasons**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**