



CONGRUENT TRIANGLES

Chapter 4



GEOMETRY 2019-20


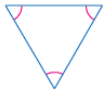
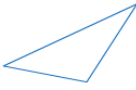

Name _____ Hour _____

Triangles can be classified by their _____ and/or their _____.

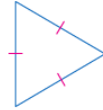
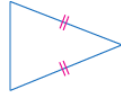
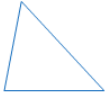
Objectives:

- 1) classify triangles by their angle measures and side lengths
- 2) use triangle classification to find angle measures and side lengths

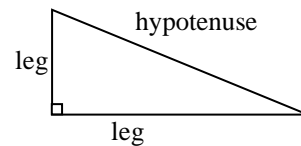
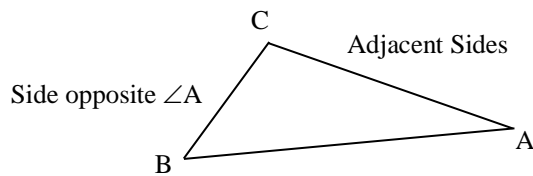
KeyConcept Classifications of Triangles by Angles

acute triangle  3 acute angles	equiangular triangle  3 congruent acute angles	obtuse triangle  1 obtuse angle	right triangle  1 right angle
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KeyConcept Classifications of Triangles by Sides

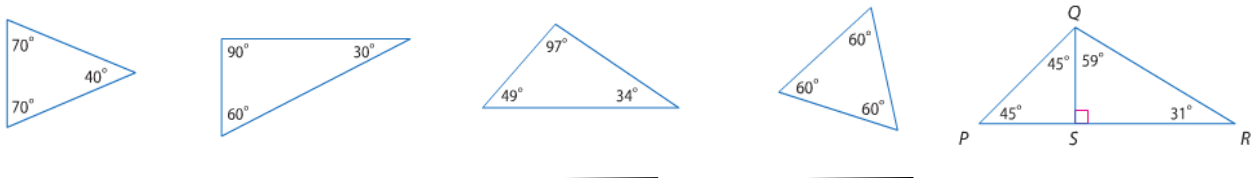
equilateral triangle  3 congruent sides	isosceles triangle  at least 2 congruent sides	scalene triangle  no congruent sides
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Triangle ABC use the symbol $\triangle ABC$



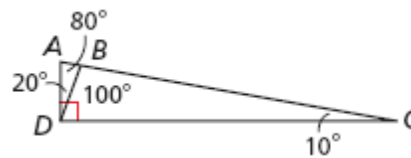
Each point is a _____.

Example 4-1-1: Classify each triangle by its angles: *acute*, *equiangular*, *obtuse*, or *right*.



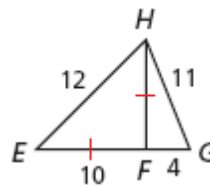
Example 4-1-2: Classify the Triangle by angle measures

- a. $\triangle BDC$
- b. $\triangle ABD$

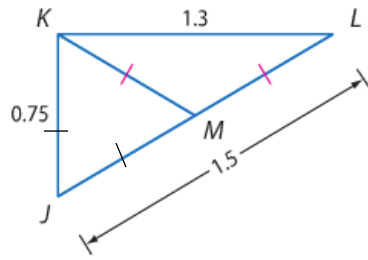


Example 4-1-3: Classify the Triangle by side lengths

- a. $\triangle EHF$
- b. $\triangle EHG$
- c. $\triangle HFG$



Example 4-1-4: If point M is the midpoint of \overline{JL} , classify $\triangle MLK$, $\triangle JKM$, and $\triangle KJL$ by their sides: *equilateral*, *isosceles*, or *scalene*.

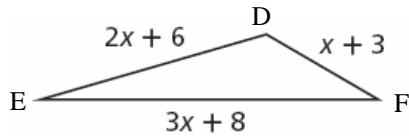


$\triangle MLK$: _____

$\triangle JKM$: _____

$\triangle KJL$: _____

Example 4-1-5: If the perimeter is 47, find x and the length of each side. $x = \underline{\hspace{1cm}}$



$DE = \underline{\hspace{1cm}}$

$DF = \underline{\hspace{1cm}}$

$EF = \underline{\hspace{1cm}}$

Why do you round the answer down instead of rounding the answer up?

Example 4-1-6: Coordinate Plane

Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

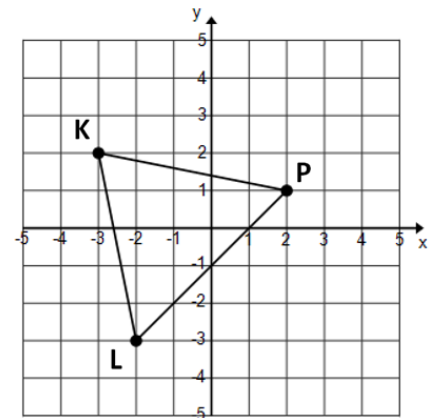
$K(-3, -2)$ $P(1, 3)$ $L(3, -3)$

$\triangle KPL$ is a _____

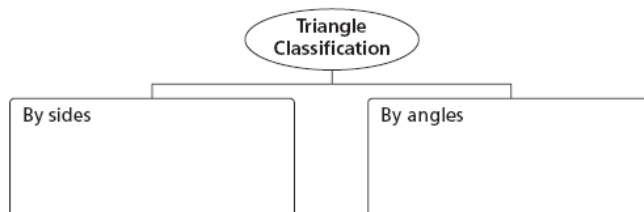
$KP =$

$PL =$

$KL =$



Think and Discuss



Auxiliary line → A line added to a diagram to help analyze the diagram. (Below: \overleftrightarrow{AD} was added to make a line parallel to the \overline{BC} by the Parallel Postulate.)

Objectives:

- 1) find the measures of interior and exterior angles of triangle
- 2) apply theorems about the interior and exterior angles of triangles

Whenever you draw an **auxiliary line**, you must be able to justify its existence. Give this as the reason: Through any two points there is exactly one line.

Proof Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

Proof:

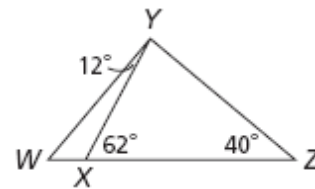
Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw \overleftrightarrow{AD} through A parallel to \overline{BC} .	2. Parallel Postulate
3. $\angle 4$ and $\angle BAD$ form a linear pair.	3. Def. of a linear pair
4. $\angle 4$ and $\angle BAD$ are supplementary.	4. If 2 \angle s form a linear pair, they are supplementary.
5. $m\angle 4 + m\angle BAD = 180$	5. Def. of suppl. \angle s
6. $m\angle BAD = m\angle 2 + m\angle 5$	6. Angle Addition Postulate
7. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	7. Substitution
8. $\angle 4 \cong \angle 1, \angle 5 \cong \angle 3$	8. Alt. Int. \angle Theorem
9. $m\angle 4 = m\angle 1, m\angle 5 = m\angle 3$	9. Def. of $\cong \angle$ s
10. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	10. Substitution

Example 4-2-1: Real World Application

After an accident, the positions of cars are measured by law enforcement to investigate the collision. Use the diagram drawn from the information collected to find the indicated angle measures.

a. $m\angle XYZ$

b. $m\angle YWZ$

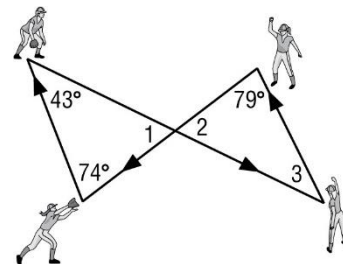


Example 4-2-2: Real World Application The diagram shows the path of the softball in a drill developed by four players. Find the measure of each numbered angle.

$m\angle 1 =$

$m\angle 2 =$

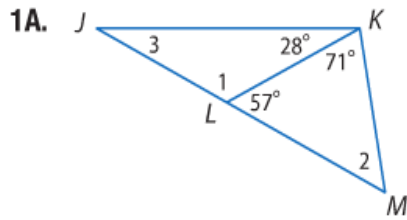
$m\angle 3 =$



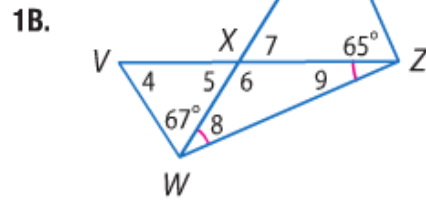
shows the path of the softball in a drill developed by four players. Find the measure of each numbered angle.

Guided Practice

Find the measures of each numbered angle.



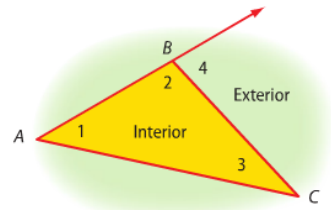
$$\begin{aligned} m\angle 1 &= \underline{\hspace{2cm}} \\ m\angle 2 &= \underline{\hspace{2cm}} \\ m\angle 3 &= \underline{\hspace{2cm}} \end{aligned}$$



$m\angle 4 =$	$m\angle 7 =$
$m\angle 5 =$	$m\angle 8 =$
$m\angle 6 =$	$m\angle 9 =$

Example 4-2-3: One of the acute angles in a right triangle measures $2x^\circ$. What is the measure of the other acute angle?

Exterior Angle \rightarrow the _____ formed by one side and a _____ of the other side
In the picture, it is angle _____



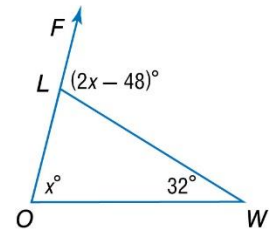
Remote Interior \rightarrow the interior angles that are not _____ to that exterior angle
In the picture, the angles are _____

Theorem 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example $m\angle A + m\angle B = m\angle 1$

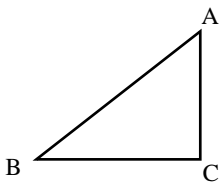
Example 4-2-4: Find the $m\angle FLW$ in the fenced flower garden shown.



$$m\angle FLW = \underline{\hspace{2cm}}$$

Corollary \rightarrow A theorem that can be proven directly from another theorem.

- Corollary 4.1 – the acute angles in a right triangle are _____
- Corollary 4.2 – there can be at most one right or obtuse angle in a triangle.
- Corollary – the angles in an Equiangular Triangle are _____.



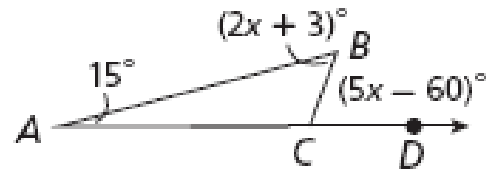
Complete the 2-column proof of Corollary 4.1:

Given: Right triangle ABC

Prove: $\angle A$ and $\angle B$ are complementary

Statements	Reasons
1. Right triangle ABC	1.
2. $m\angle A + m\angle B + m\angle C = 180$	2.
3. $m\angle C = 90$	3. Def. of right triangle
4. $m\angle A + m\angle B + 90 = 180$	4.
5.	5. Subtraction
6. $\angle A$ and $\angle B$ are complementary	6.

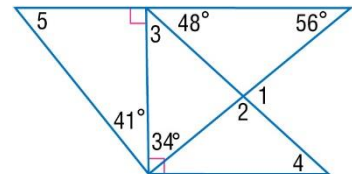
Example 4-2-5: Find the $m\angle B$.



$x =$ _____

$m\angle B =$ _____

Example 4-2-6: Find Angle Measures in Right Triangles



$m\angle 1 =$	$m\angle 2 =$	$m\angle 3 =$
$m\angle 4 =$	$m\angle 5 =$	

Think and Discuss

THEOREM	WORDS	DIAGRAM
Triangle Sum Theorem		
Exterior Angles Theorem		
Third Angles Theorem		

Objectives:

1. use properties of congruent triangles
2. prove triangles congruent by using the definition of congruence

Two figures are _____ when they have corresponding angles and corresponding sides that are congruent.

KeyConcept Definition of Congruent Polygons

Words	Two polygons are congruent if and only if their corresponding parts are congruent.	Model
Example	<p>Corresponding Angles $\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$</p> <p>Corresponding Sides $\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$</p> <p>Congruence Statement $\triangle ABC \cong \triangle HJK$</p>	

Theorem 4.4 Properties of Triangle Congruence

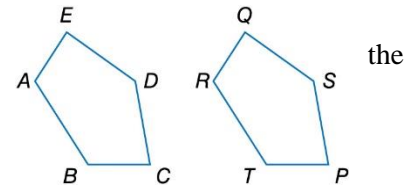
Reflexive Property of Triangle Congruence
 $\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence
 If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence
 If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Example 4-3-1: Identify Corresponding Congruent Parts

Show that the polygons are congruent by identifying all of congruent corresponding parts. Then write a congruence statement.



Sides	Angles
Congruence statement:	

Triangle Congruence (CPCTC)

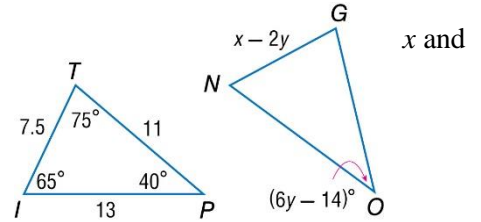
Knowing that all pairs of corresponding parts of congruent triangles are congruent (CPCTC) can help you reach conclusions about congruent figures.

CPCTC → Corresponding Parts of Congruent Triangles are Congruent

CPCTC → Corresponding Parts of Congruent Triangles are Congruent

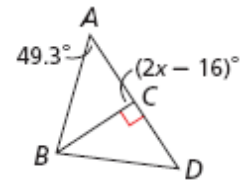
Example 4-3-2: Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle ITP \cong \triangle NGO$. Find the values of x and y .



Example 4-3-3: Given: $\triangle ABC \cong \triangle DBC$

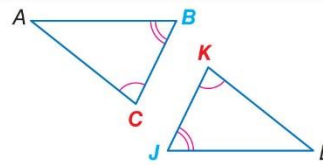
- Find the value of x
- Find the $m\angle DBC$



Theorem 4.3 Third Angles Theorem

Words: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

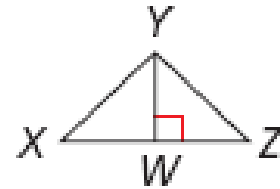
Example: If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



Example 4-3-4: Proving triangles congruent.

Given: $\angle YWX$ and $\angle YWZ$ are right angles. \overline{YW} bisects $\angle XYZ$. W is the midpoint of \overline{XZ} . $\overline{XY} \cong \overline{YZ}$

Prove: $\triangle XYW \cong \triangle ZYW$



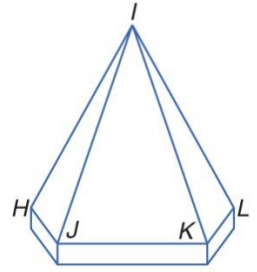
Definition of congruent figures:

In order to prove the triangles congruent we need to show all the corresponding sides and angles are congruent.

Statements	Reasons
1. $\angle YWX$ and $\angle YWZ$ are right angles	Given
2. W is the midpoint of \overline{XZ} .	Given
3.	Def. of midpoint
4. \overline{YW} bisects $\angle XYZ$	Given
5.	Def. of \angle bisector
6.	All right \angle 's \cong
7. $\overline{XY} \cong \overline{YZ}$	Given
8.	Reflexive
9.	Third \angle 's Thm
10. $\triangle XYW \cong \triangle ZYW$	

Example 4-3-5: Use the Third Angles Theorem

A drawing of a tower's roof is composed of congruent triangles all converging at a point at the top. If $\angle IJK \cong \angle IKJ$ and $m\angle IJK = 72$, find $m\angle JIH$.

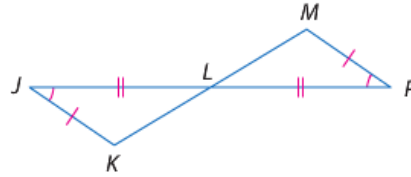


Example 4-3-6: Prove Triangles Congruent

Given: $\overline{JL} \cong \overline{PL}$, and L bisects \overline{KM} .

$\angle J \cong \angle P, \overline{JK} \cong \overline{PM}$

Prove: $\triangle JLK \cong \triangle PLM$



Statements	Reasons
1 $\overline{JL} \cong \overline{PL}, \angle J \cong \angle P, \overline{JK} \cong \overline{PM}$	1. Given
2 L bisects \overline{KM}	2. Given
3 $\overline{KL} \cong \overline{LM}$	3.
4	4. Vertical Angles Thm
5 $\angle K \cong \angle M$	5.
6 $\triangle JLK \cong \triangle PLM$	6.

Think and Discuss – complete the graphic organizer



Objectives:

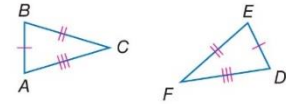
1. apply SSS and SAS to construct triangles and to solve problems
2. prove triangles congruent by using SSS and SAS to construct triangles and to solve problems.

Triangle Rigidity –

Postulate 4.1 Side-Side-Side (SSS) Congruence

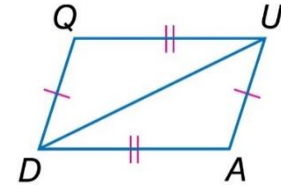
If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If Side $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.



Example 4-4-1: Use SSS to Prove Triangles Congruent

Given: $\overline{QU} \cong \overline{AD}$, $\overline{QD} \cong \overline{AU}$
Prove: $\triangle QUD \cong \triangle ADU$

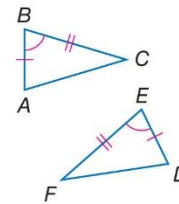


Statements	Reasons
1. $\overline{QU} \cong \overline{AD}$, $\overline{QD} \cong \overline{AU}$	1. Given
2.	2.
3. $\triangle QUD \cong \triangle ADU$	3.

Postulate 4.2 Side-Angle-Side (SAS) Congruence

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

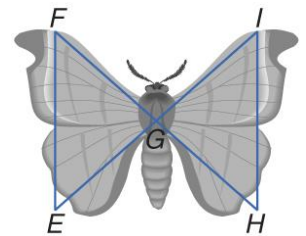
Example If Side $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Included Angle →

Example 4-4-2: Use SAS to Prove Triangles are Congruent

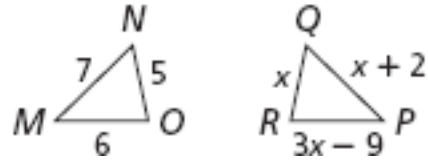
The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$, if $\overline{EI} \cong \overline{FH}$, and G is the midpoint of both \overline{EI} and \overline{FH} .



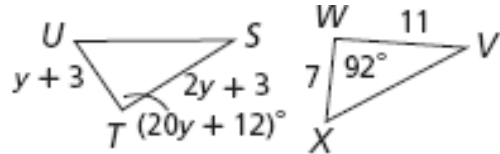
Statements	Reasons
1. $\overline{EI} \cong \overline{FH}$	1. Given
2. G is the midpoint of both \overline{EI} and \overline{FH}	2. Given
3. $\overline{EG} \cong \overline{GI}$, $\overline{FG} \cong \overline{GH}$	3.
4. $\angle FGE \cong \angle IGH$	4.
5. $\triangle FEG \cong \triangle HIG$	5.

Example 4-4-4: Show that the triangles are congruent for the given value of the variable.

a. $\triangle MNO \cong \triangle PQR$, when $x = 5$



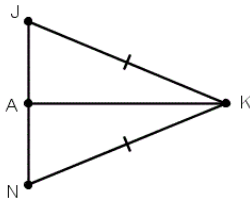
b. $\triangle STU \cong \triangle VWX$, when $y = 4$



Example 4-4-5: Determine if SSS or SAS or neither is used in the following triangle congruency. Mark your diagram!

Ex A:

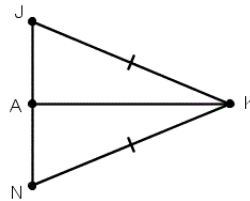
Given: A is the midpoint of \overline{JN}



SSS SAS Neither

Ex B:

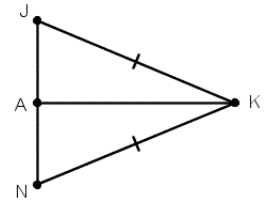
Given: \overline{KA} bisects $\angle JKN$



SSS SAS Neither

Ex C:

Given: $\angle J \cong \angle N$

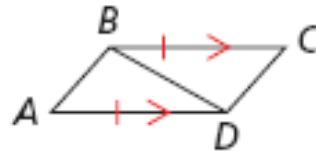


SSS SAS Neither

Example 4-4-6:

Given: $\overline{BC} \parallel \overline{AD}$, $\overline{BC} \cong \overline{AD}$

Prove: $\triangle ABD \cong \triangle CDB$

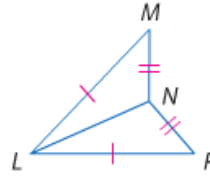


Statements	Reasons
1. $\overline{BC} \parallel \overline{AD}$	1. Given
2. $\overline{BC} \cong \overline{AD}$	2. Given
3.	3.
4.	4.
5. $\triangle ABD \cong \triangle CDB$	5.

Example 4-4-7:

Given: $\overline{MN} \cong \overline{PN}, \overline{LM} \cong \overline{LP}$

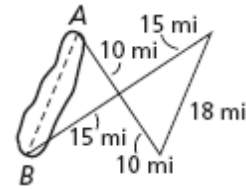
Prove: $\angle LNM \cong \angle LNP$



Statements	Reasons
1. $\overline{MN} \cong \overline{PN}, \overline{LM} \cong \overline{LP}$	1. Given
2.	2.
3. $\triangle LMN \cong \triangle LPN$	3.
4. $\angle LNM \cong \angle LNP$	4. CPCTC

Example 4-4-8: Real World Application

A and B are on the edges of a ravine. What is AB?



Geometry	Proving Triangles are Congruent: ASA and AAS	4.5
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Objectives:

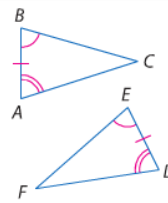
Included side →

1. apply ASA, AAS, and HL to construct triangles and to solve problems
2. prove triangles congruent by using ASA, AAS, and HL to construct triangles and to solve problems.

Postulate 4.3 Angle-Side-Angle (ASA) Congruence

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

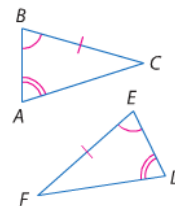
Example If $\angle A \cong \angle D$,
 $\text{Side } \overline{AB} \cong \overline{DE}$, and
 $\angle B \cong \angle E$,
 then $\triangle ABC \cong \triangle DEF$.



Theorem 4.5 Angle-Angle-Side (AAS) Congruence

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

Example If $\angle A \cong \angle D$,
 $\angle B \cong \angle E$, and
 $\text{Side } \overline{BC} \cong \overline{EF}$,
 then $\triangle ABC \cong \triangle DEF$.

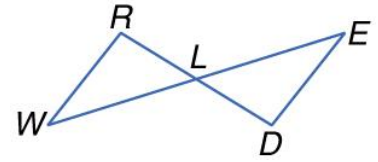


NOTE: Two methods that **CANNOT** be used are: AAA and SSA

Example 4-5-1 – Use ASA to Prove Triangles Congruent

Think about
Example 4-5-1

Given: $\overline{WR} \parallel \overline{ED}$, L is the midpoint of \overline{WE}
Prove: $\triangle WRL \cong \triangle EDL$



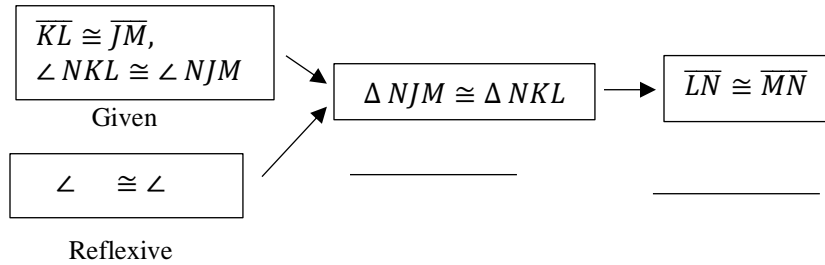
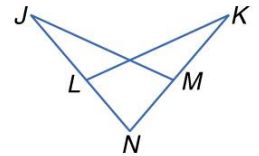
We used ASA, can it be done with AAS?
If not, why not.

If so, what additional information would you need?

Statements	Reasons
1. $\overline{WR} \parallel \overline{ED}$, L is the midpoint of \overline{WE}	1. Given
2.	2. def of midpoint
3.	3.
4. $\angle \cong \angle$	4.
5. $\triangle WRL \cong \triangle EDL$	5.

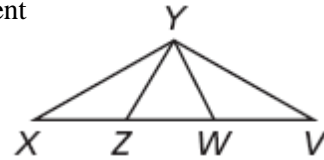
Example 4-5-2 – Use AAS to Prove Triangles Congruent

Given: $\overline{KL} \cong \overline{JM}$, $\angle NKL \cong \angle NJM$
Prove: $\overline{LN} \cong \overline{MN}$



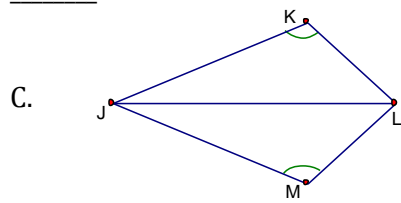
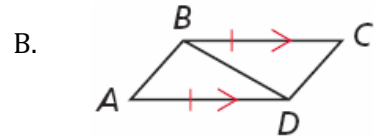
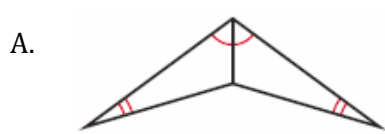
Example 4-5-3: Use AAS to prove the triangles congruent

Given: $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$
Prove: $\triangle XYZ \cong \triangle VYW$



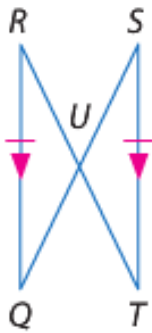
Statements	Reasons
1. $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$	1. Given
2. $\triangle XYW \cong \triangle VYZ$	2.
3. $\overline{YZ} \cong \overline{YW}$	3.
4. $\angle \cong \angle$	4. Congruent Supplements Thm.
5. $\triangle XYZ \cong \triangle VYW$	5.

Example 4-5-4: Which method would you use? Support your reasoning with correct markings on the diagram.



Given: \overline{JL} bisects $\angle KLM$. $\angle K \cong \angle M$.

Example 4-5-5: Flow Proof



Given: $\overline{RQ} \parallel \overline{ST}$, $\overline{RQ} \cong \overline{ST}$
Prove: $\triangle RUQ \cong \triangle TUS$

Plan 1

$\overline{RQ} \parallel \overline{ST}$,
 $\overline{RQ} \cong \overline{ST}$

$\angle R \cong \angle T$
 $\angle Q \cong \angle S$

$\triangle RUQ \cong \triangle TUS$

Plan 2

$\overline{RQ} \parallel \overline{ST}$,
 $\overline{RQ} \cong \overline{ST}$

$\angle R \cong \angle T$

$\angle RUQ \cong \angle SUT$

$\triangle RUQ \cong \triangle TUS$

Plan: 1st possible approach: Use parallel line theorem twice to use ASA.

2nd possible approach: Use a parallel line theorem and vertical angles to use AAS.

4.5 – Extension: Right Triangle Congruence: HL

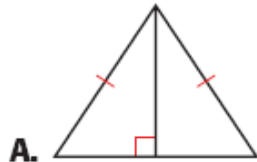
Theorem 4.9 Hypotenuse-Leg Congruence

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

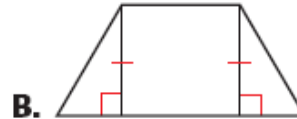


Abbreviation HL

Example 4-5-6: Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.



Yes _____ No: _____

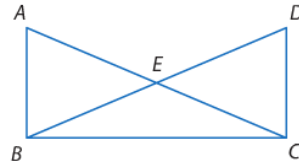


Yes _____ No: _____

Example 4-5-7:

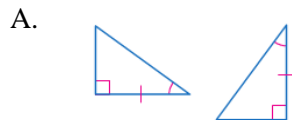
Given: $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}, \overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{DC}$

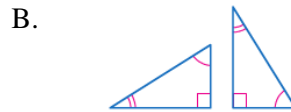


Statements	Reasons
1. $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2.
3.	3. Def. of \perp
4.	4. All right \angle 's \cong
5. $\overline{AC} \cong \overline{BD}$	5. Given
6. $\triangle ABC \cong \triangle DCB$	6.
7. $\overline{AB} \cong \overline{DC}$	7.

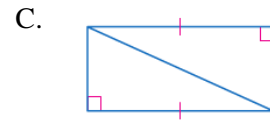
Example 4-5-8: Determine whether each pair of triangles is congruent. If yes, tell which postulate or theorem applies.



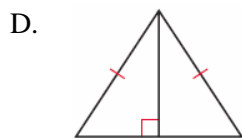
Yes: _____ No



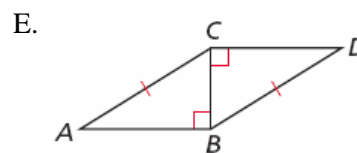
Yes: _____ No



Yes: _____ No



Yes: _____ No



Yes: _____ No

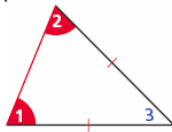
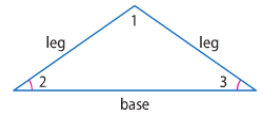
Objectives:

1. prove theorems about isosceles and equilateral triangles
2. apply properties of isosceles and equilateral triangles

Isosceles triangle → An **isosceles triangle** has two congruent sides called the **legs**. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

$\angle 1$ is the vertex angle.

$\angle 2$ and $\angle 3$ are the base angles.



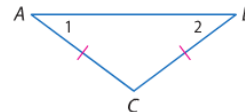
In this diagram, the base angles are _____ and _____; the vertex angle is _____.

Theorems Isosceles Triangle

4.10 Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

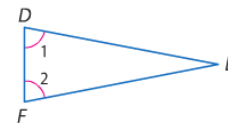
Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



4.11 Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

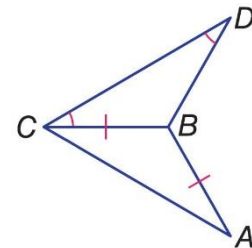
Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.



Example 4-6-1: **Congruent Segments and Angles**

A. Name two unmarked congruent angles.

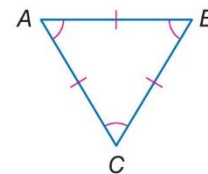
B. Name two unmarked congruent segments.



Corollaries Equilateral Triangle

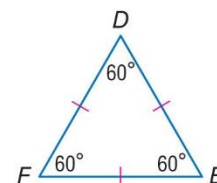
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

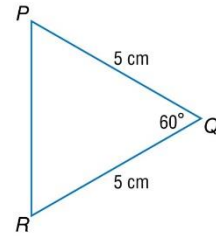
Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.



Example 4-6-2: **Find Missing Measures**

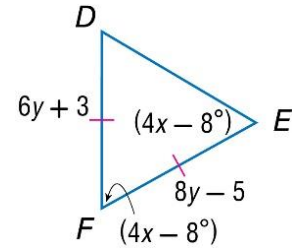
A. Find $m\angle R$.

B. Find PR .

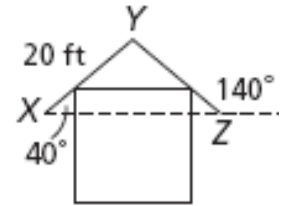


Example 4-6-3: **Find Missing Values**

Find the value of each variable.

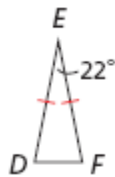


Example 4-6-4: The length of \overline{YX} is 20 feet. Explain why the length of \overline{YZ} is the same.



Example 4-6-5: Find each angle measure.

a. $m\angle F$

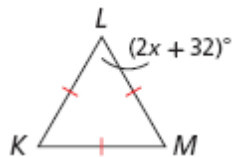


b. $m\angle G$

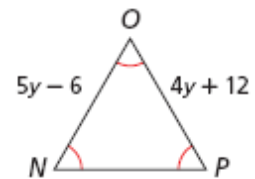


Example 4-6-6: – Find each value.

a. x

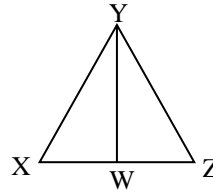


b. y



Example 4-6-8:

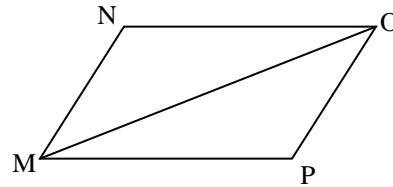
Given: \overline{YW} bisects \overline{XZ} $\overline{XY} \cong \overline{YZ}$
 Prove: $\angle XYW \cong \angle ZYW$



Statements	Reasons
1. \overline{YW} bisects \overline{XZ} $\overline{XY} \cong \overline{YZ}$	1. Given
2. $\overline{XW} \cong \overline{WZ}$	2.
3. $\overline{YW} \cong \overline{YW}$	3.
4. $\triangle XYW \cong \triangle ZYW$	4.
5. $\angle XYW \cong \angle ZYW$	6.

Example 4-6-9: Using CPCTC with another Thm.

Given: $\overline{NO} \parallel \overline{MP}$, $\angle N \cong \angle P$
 Prove: $\overline{MN} \parallel \overline{OP}$



Statements	Reasons
1. $\overline{NO} \parallel \overline{MP}$, $\angle N \cong \angle P$	1. Given
2. $\angle NOM \cong \angle PMO$	2.
3. $\overline{MO} \cong \overline{MO}$	3.
4. $\triangle MNO \cong \triangle OPM$	4.
5. $\angle NMO \cong \angle POM$	5.
6. $\overline{MN} \parallel \overline{OP}$	6.

	Def of $\triangle \cong$	SSS	SAS	ASA	AAS	HL
Words	All \angle 's \cong All sides \cong	All 3 sides \cong	2 sides \cong & Included angle \cong	2 \angle 's and included side \cong	2 \angle 's and un-included side \cong	Hypotenuse Leg in a right \triangle
Pictures						

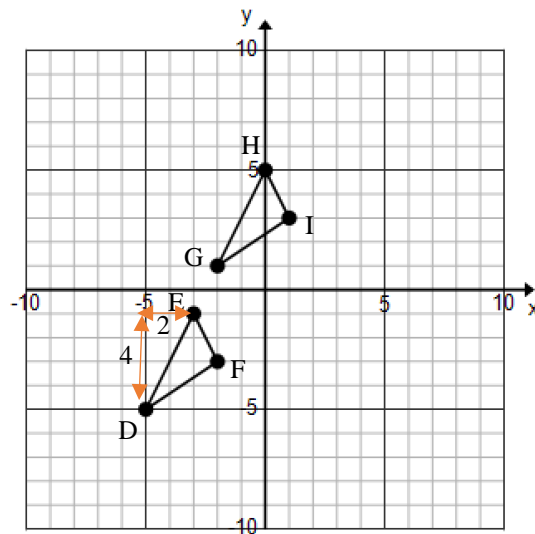
Objectives:

1. Position and label triangles for use in coordinate proofs.
2. Write coordinate proofs.

Example 4-8-1: Proof in the Coordinate Plane

Given: $D(-5, -5), E(-3, -1), F(-2, -3),$
 $G(-2, 1), H(0, 5)$ and $I(1, 3)$

Prove: $\angle DEF \cong \angle GHI$



$DE = \sqrt{(-1 - -5)^2 + (-3 - -5)^2}$ Or $\sqrt{(4)^2 + (2)^2} = \underline{\hspace{2cm}}$
$EF =$
$FD =$
$GH =$
$HI =$
$IG =$

Apply the **distance formula** to all 3 sides of each triangle:

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

or

$$d = \sqrt{(leg)^2 + (leg)^2}$$

a) $\triangle XYW \cong \triangle ZYW$? Yes No

If yes, how? _____

b) $\angle DEF \cong \angle GHI$ by _____

Strategies for doing Coordinate Proof

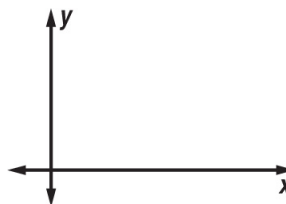
1. Use origin as a vertex
2. center figure at the origin
3. center side of figure at origin
4. use axes as sides of figure

KeyConcept Placing Triangles on Coordinate Plane

- Step 1** Use the origin as a vertex or center of the triangle.
- Step 2** Place at least one side of a triangle on an axis.
- Step 3** Keep the triangle within the first quadrant if possible.
- Step 4** Use coordinates that make computations as simple as possible.

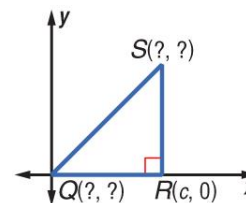
Example 4-8-2: **Position and Label a Triangle**

Position and label right triangle XYZ with leg d units long on the coordinate plane.



Example 4-8-3: **Identify Missing Coordinates**

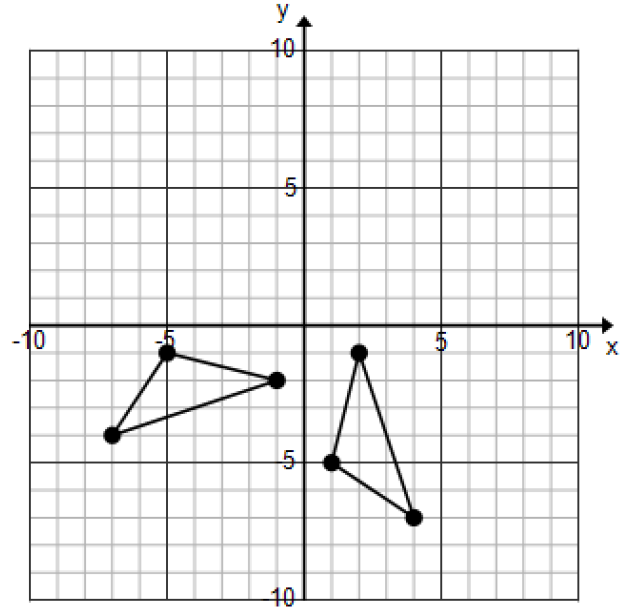
Name the missing coordinates of isosceles right triangle QRS .



Example 4-8-4: SSS on the Coordinate Plane

Triangle DVW has vertices $D(-5, -1)$, $V(-1, -2)$, and $W(-7, -4)$. Triangle LPM has vertices $L(1, -5)$, $P(2, -1)$, and $M(4, -7)$.

- Graph both triangles on the same coordinate plane.
- Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.
- Write a logical argument that uses coordinate geometry to support the conjecture you made in part b.



Apply the **distance formula** to all 3 sides of each triangle:

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \text{ or } d = \sqrt{(leg)^2 + (leg)^2}$$

$DV = \sqrt{(-1 - -5)^2 + (-2 - -1)^2}$ or $d = \sqrt{(5)^2 + (1)^2}$
$VW =$
$WD =$
$LP =$
$PM =$
$ML =$