TRIANGLE RELATIONSHIPS

Chapter 5

Name ______________________________

Hour ________________

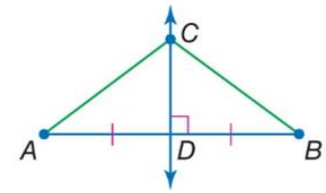
Geometry

Objectives:

1. Identify and use perpendicular bisectors in triangles.
2. Identify and use angle bisectors in

The Perpendicular Bisector Theorem:
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment

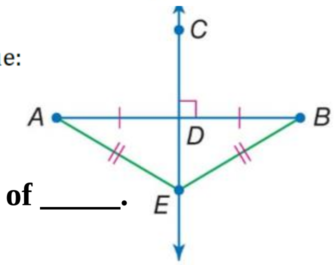
Example: If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then **AC = ______**



The Converse of the Perpendicular Bisector Theorem is also true:
If a point is equidistant from the endpoints of a segment, then

it is on the perpendicular bisector of the segment.

Example: If $AE = BE$, then ____ **lies on $\overline{CD}$, *the* $\perp$ bisector of _____.**



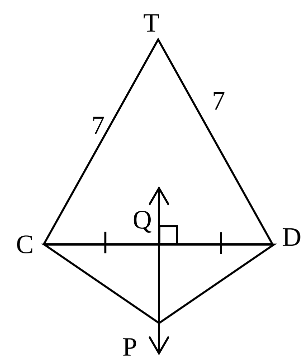
Perpendicular Bisector - is a segment or LINE that cuts a SEGMENT into two equal pieces. In a triangle, it may or may not go through the vertex of the opposite angle.
Equidistant from two points equal distance from two points
Distance from a point to a line (p. 215) - the length of the segment perpendicular to the line from the point.
Concurrent Lines (or rays or segments) - three or more lines intersect at a common point
Point of Concurrency - the point where the three or more lines intersect.

Example 5-1-1:

In the diagram shown, $\overleftrightarrow{PQ}$ is the perpendicular bisector $\overline{CD}$.

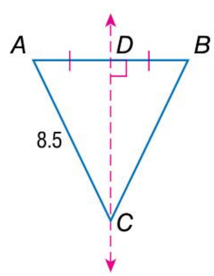
a. What segment lengths in the diagram are equal?

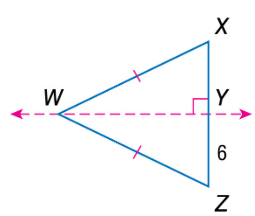
b. Explain why T is on $\overleftrightarrow{PQ}$.

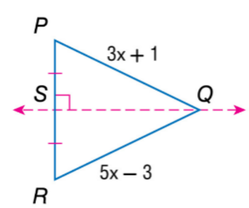


Example 5-1-2: **Use the Perpendicular Bisector Theorems**

A. Find *BC*=_____ B. Find *XY*=_____ C. Find *PQ*= ______





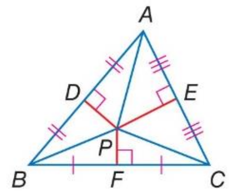


__________ point of intersection forms the __________ of the triangle.

Theorem 5.3 Circumcenter Theorem

Words The perpendicular bisectors of a triangle intersect at a point called the *circumcenter* that is equidistant from the vertices of the triangle.

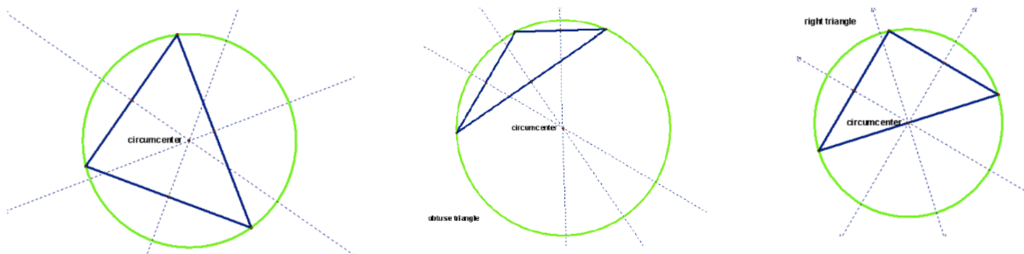
Example If P is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.



The <u>circumcenter</u> of a triangle is equidistant from the

________________ of the triangle.

The circumcenter can be on the interior, exterior, or side of a triangle.

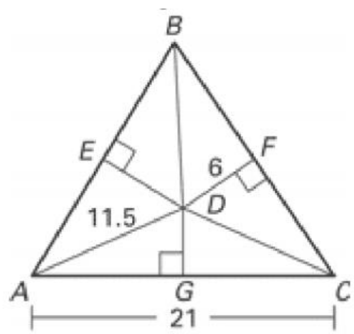


Example 5-1-3: **Use the Circumcenter Theorem**

The perpendicular bisectors of $\Delta\, ABC$ meet at point D.

BD = _____

DC = _____



Example 5-1-4: Using Circumcenter

Three people need to decide on a location to hold a monthly meeting. They will all be coming from different places in the city and they want to make the meeting location the same distance for each person. Explain why using circumcenter as the location for the meeting would be fairest for all.

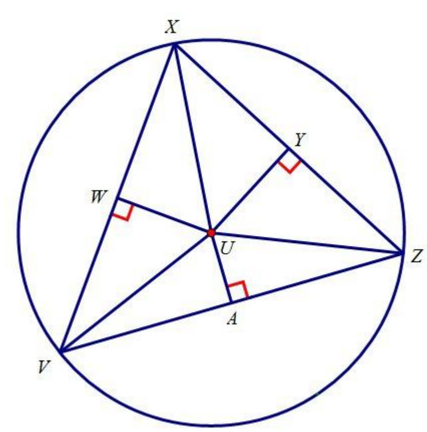
Example 5-1-5: Using Circumcenter

$\overline{VW} \cong$ _____

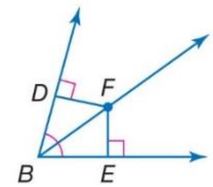
$\overline{AZ} \cong$ _____

$\overline{XY} \cong$ _____

$\overline{XU} \cong \overline{VU} \cong$ _____



Angle Bisector Theorem: If a point is on the bisector of an angle, then



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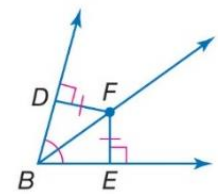
Example: If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$ and $\overline{FE} \perp \overrightarrow{BE}$ then, DF = _____

The Converse of the Angle Bisector Theorem is also true: If a point in the interior of an angle is equidistant from the sides of an angle, then



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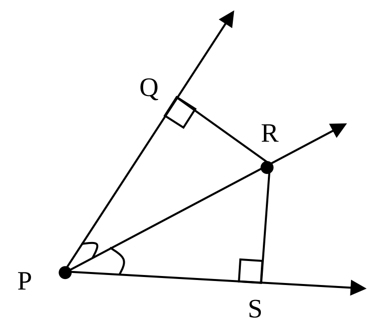
Example: If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and DF = FE, then $\overrightarrow{BF}$ bisects $\angle$ ______

Example 5-1-6:
Given: R is on the bisector of $\angle QPS$.
$\overline{RQ} \perp \overline{PQ}, \overline{RS} \perp \overline{PS}$
Prove: $\overline{RQ} \cong \overline{RS}$



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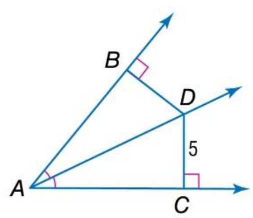
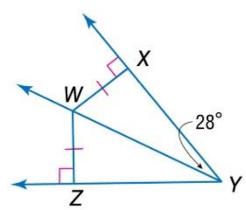
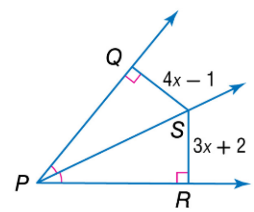
Example 5-1-7: **Use the Angle Bisector Theorems**

A. Find DB= ______ B. Find $m\angle WYZ$ = ______ C. Find QS = _______







USING PROPERTIES OF PERPENDICULAR AND ANGLE BISECTORS

Perpendicular Bisector of a Triangle - is a segment or line that cuts a SEGMENT into two equal pieces. In a triangle, it may or may not go through the vertex of the opposite angle.
Circumcenter - A point equidistant from the vertices of the triangle, which allows you to circumscribe the triangle with a circle.
Angle Bisector of a Triangle - A ray that bisects the angle in a triangle.
(may or may not go through the midpoint of the opposite side.)
Incenter of a Triangle - A point equidistant from the sides of the triangle, which allows you to inscribe a circle inside the triangle.

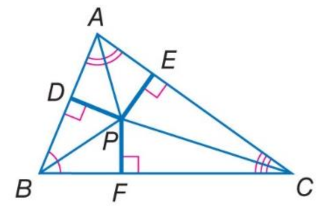
_____ __________

point of intersection forms the __________ of the triangle. This point is ___________ from the sides of the triangle.

Theorem 5.6 Incenter Theorem

Words The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle.

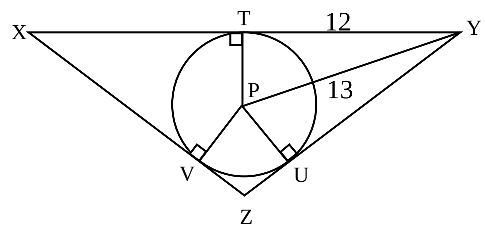
Example If *P* is the incenter of △*ABC*, then *PD* = *PE* = *PF*.



Example 5-1-7: Using Angle Bisectors

The angle bisector of △ XYZ meet at point P.

a. What segments are congruent?

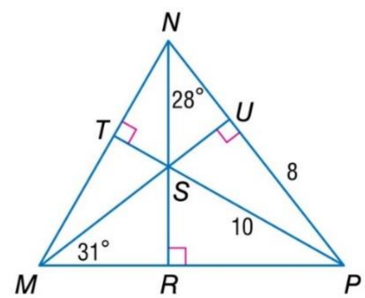


b. Find PT= _____ c. PV= _____

Example 5-1-8: **Use the Incenter Theorem**

A. Find *SU* if *S* is the incenter of Δ*MNP*.

B. Find *m*∠*SPU* if *S* is the incenter of Δ*MNP*.

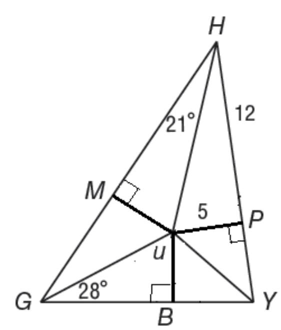


Example 5-1-9: **Use the Incenter Theorem**

Point *U* is the incenter of △*GHY*. Find each measure.

a. *MU* **b.** *m*∠*UGM*

c. *m*∠*PHU* **d.** *HU*



Objectives:

1. Identify and use medians in triangles.
2. Identify and use altitudes in triangles.

__________ point of intersection forms the __________ of the triangle. This point is ___________ from the vertex to the midpoint of the opposite side of the triangle.

5-2 Medians and Altitudes of Triangles

Median of a Triangle - A segment that connects the midpoint of a side with the vertex of the opposite angle.

Centroid of a Triangle - The point where the medians intersect. It is the center of gravity for the triangle.

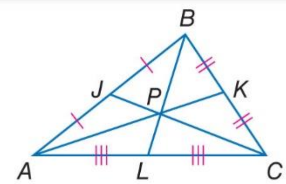
Altitude of a Triangle - A segment from a vertex, that is perpendicular to the opposite side (which may be extended outside the triangle.)

Orthocenter of the Triangle The point where the altitudes intersect.

Theorem 5.7 Centroid Theorem

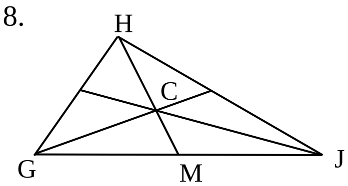
The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

Example If P is the centroid of $\triangle ABC$, then $AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.



Example 5-2-1: **Use the Centroid Theorem**

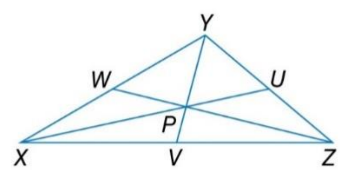
a. C is the centroid of $\triangle GHJ$ and CM = 8.



HM = _____

CH = _____

b. In ΔXYZ, P is the centroid and $YV = 12$. Find YP and PV.

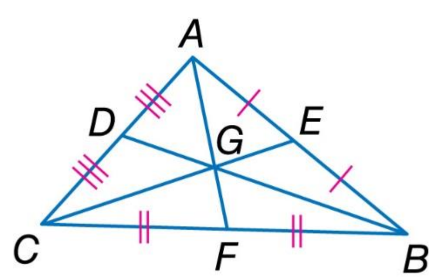


YP = _____

PV = _____

Example 5-2-2: **Use the Centroid Theorem**

In ΔABC, $CG = 6$. GF = 2, FB = 7



GE = _____

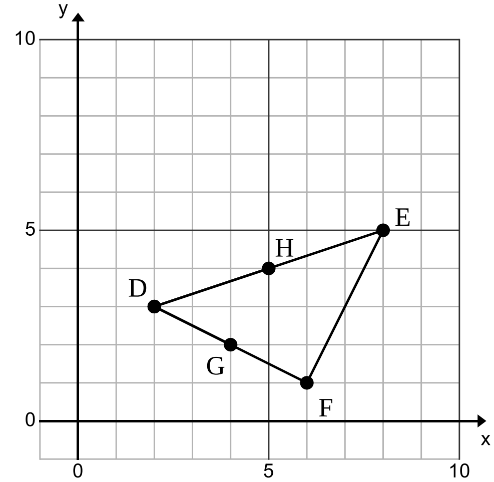
AG = _____

CF = _____

Example 5-2-3: **Find the Centroid on a Coordinate Plane**

Find the coordinate of the centroid P of ΔDEF.

D (2, 3) E (8, 5) F (6, 1)
G (4, 2) *H (5, 4)*



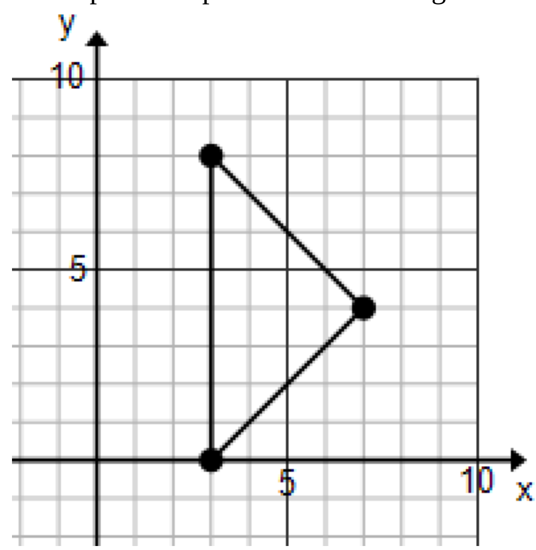
Segments EG and HF point of intersection is the centroid.

Another formula for centroid is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Example 5-2-4: **Find the Centroid on a Coordinate Plane**

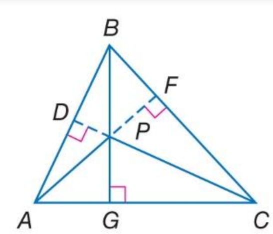
An artist is designing a sculpture that balances a triangle on top of a pole. In the artist's design on the coordinate plane, the vertices are located at $(7, 4)$, $(3, 0)$, and $(3, 8)$. What are the coordinates of the point where the artist should place the pole under the triangle so that it will balance?



KeyConcept Orthocenter

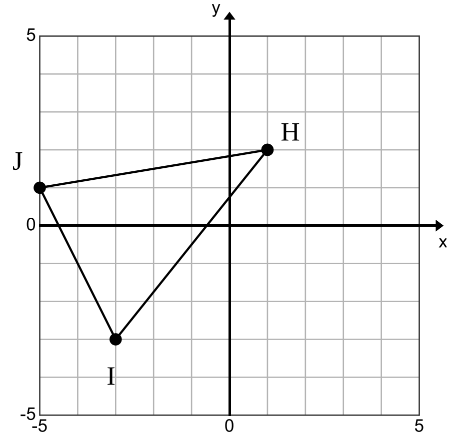
The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the **orthocenter**.

Example The lines containing altitudes $\overline{AF}$, $\overline{CD}$, and $\overline{BG}$ intersect at *P*, the orthocenter of $\triangle ABC$.



__________ point of intersection forms the __________ of the triangle.

Example 5-2-5: **Find the Orthocenter on a Coordinate Plane**
The vertices of Δ*HIJ* are *H*(1, 2), *I*(–3, –3), and *J*(–5, 1). Find the coordinates of the orthocenter of Δ*HIJ*.



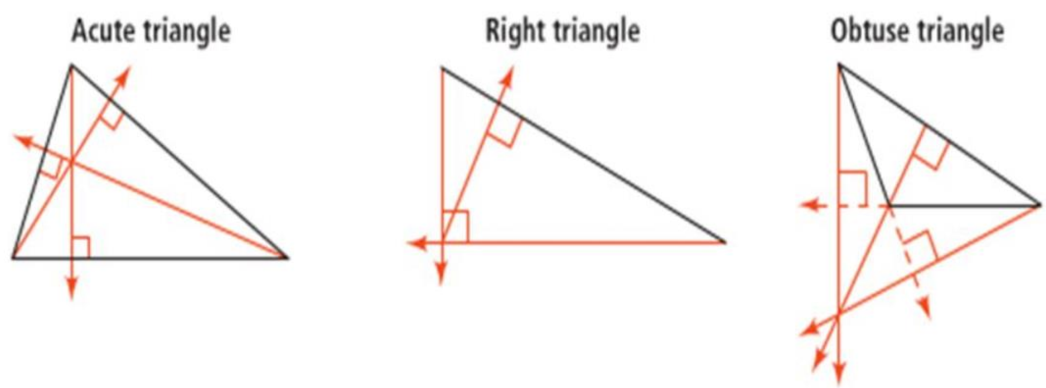
Slope of $\overleftrightarrow{JH}$ = ____ ⊥ Slope = ____ (go from I)

Slope of $\overleftrightarrow{JI}$ = ____ ⊥ Slope = ____ (go from H)

Slope of $\overleftrightarrow{IH}$ = ____ ⊥ Slope = ____ (go from J)

Example 5-2-6: Orthocenter

Where is the orthocenter located in $\Delta\ ABC$? Is it ***inside, outside*** or ***on*** the triangle?



Section Summary

Determine a way to remember each of the different points of concurrency, how they are found and what they mean. Then describe your method here.

Circumcenter - perpendicular bisectors →

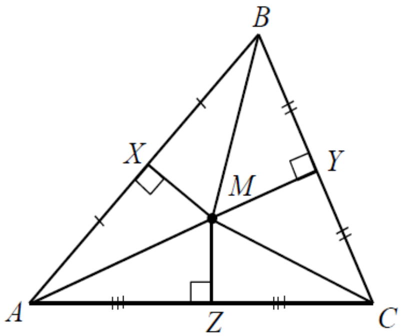
Incenter – angle bisectors →

Centroid - Medians →

Orthocenter - Altitudes →

Centers of Triangles

Circumcenter



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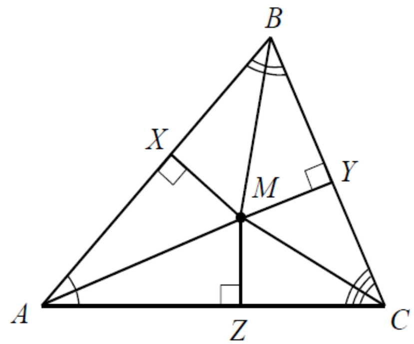
Important Facts:

The ______________________ is equidistant from each __________ of the triangle.

Incenter



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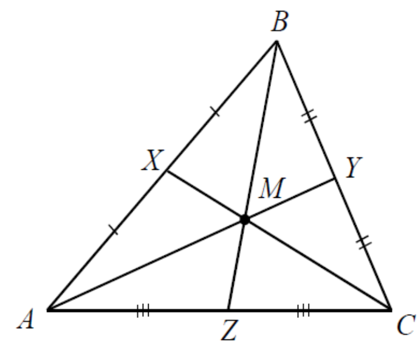
Important Facts:

The ____________________ is equidistant from each _________ of the triangle.

Centroid



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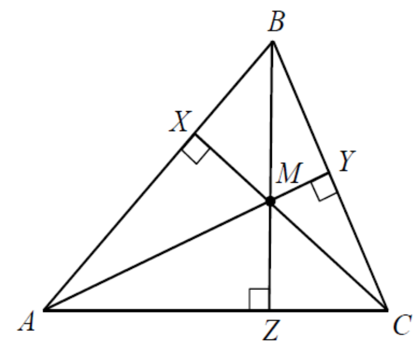
Important Facts:

A _______________ is created by a _____________ connected to the _____________ of the opposite side.

Orthocenter



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Important Facts:

An __________________ is created by a vertex connected to the opposite side so that it is

____

to that side.

Objectives:

1. Recognize and apply properties of inequalities to the measures of the angles of a triangle.
2. Recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

KeyConcept Definition of Inequality

Words	For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.
Example	If $5 = 2 + 3$, then $5 > 2$ and $5 > 3$.

KeyConcept Properties of Inequality for Real Numbers

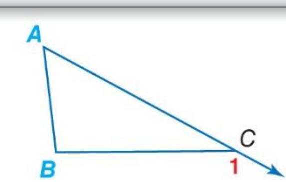
The following properties are true for any real numbers a, b, and c.

Comparison Property of Inequality	$a < b$, $a = b$, or $a > b$
Transitive Property of Inequality	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition Property of Inequality	**1.** If $a > b$, then $a + c > b + c$. **2.** If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality	**1.** If $a > b$, then $a - c > b - c$. **2.** If $a < b$, then $a - c < b - c$.

Theorem 5.8 Exterior Angle Inequality

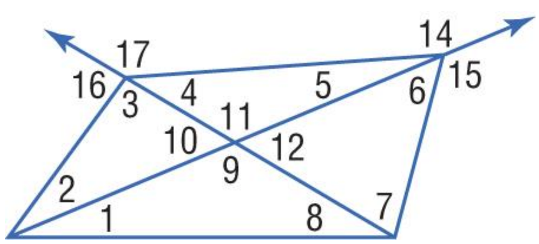
The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

Example: $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



Example 5-3-1: **Use the Exterior Angle Inequality Theorem**

Consider each list of angles, and determine which angle in the list is largest and which is the smallest.



$\angle 2, \angle 10, \angle 16$

$\angle 14, \angle 11, \angle 4$

$\angle 1, \angle 8, \angle 10$

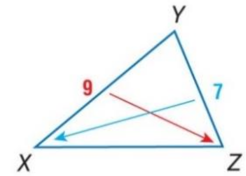
$\angle 11, \angle 5, \angle 17$

$\angle 7, \angle 15, \angle 12$

Theorems Angle-Side Relationships in Triangles

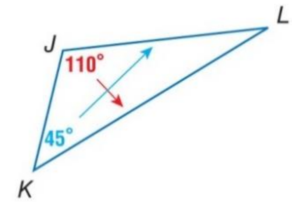
5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Example: $XY > YZ$, so $m\angle Z > m\angle X$.



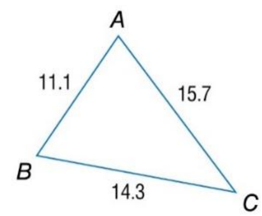
5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Example: $m\angle J > m\angle K$, so $KL > JL$.

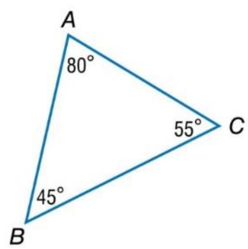


Example 5-3-2: **Order Triangle Angle Measures**

List the angles of ΔABC in order from smallest to largest.

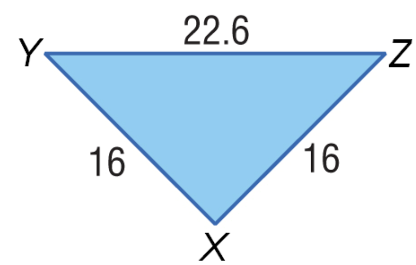


Example 5-3-3: **Order Triangle Side Lengths**



List the sides of ΔABC in order from shortest to longest.

Example 5-3-4: **Angle-Side Relationships**
Ebony is following directions for folding a handkerchief to make a bandana for her hair. After she folds the handkerchief in half, the directions tell her to tie the two smaller angles of the triangle under her hair. If she folds the handkerchief with the dimensions shown, which two ends should she tie?



ACT Example 5-3-5: Jamie claims, "If a triangle is in Set A, then it is not isosceles." Later, Jamie discovers that ΔMNP is a counterexample proving this claim false. Which of the following statements *must* be true about ΔMNP?

A. It is isosceles and in Set A
B. It is scalene and in Set A
C. It is obtuse and not in Set A
D. It is scalene and not in Set A
E. It is isosceles and not in Set A

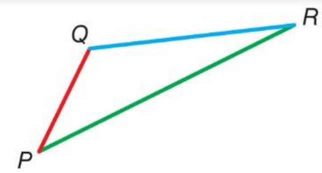
Objectives:

1. Use the Triangle Inequality Theorem to identify possible triangles.
2. Prove triangle relationships using the Triangle Inequality Theorem.

Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Examples $PQ + QR > PR$
$QR + PR > PQ$
$PR + PQ > QR$



Example 5-5-1: Constructing a Triangle

Construct a triangle with the given side lengths, if possible.

a. 3 in, 3 in, 8 in

b. 6 in, 6in, 12 in

c. 9 in, 5 in, 11 in

Example 5-5-2: Identify Possible Triangles Given Side Lengths

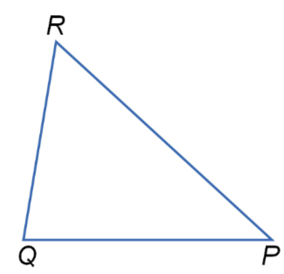
A. Is it possible to form a triangle with side lengths of $6\frac{1}{2}, 6\frac{1}{2}$ and $14\frac{1}{2}$? If not, explain why not.

B. Is it possible to form a triangle with side lengths of 6.8, 7.2 and 5.1? If not, explain why not.

Example 5-5-3: Find Possible Side Lengths

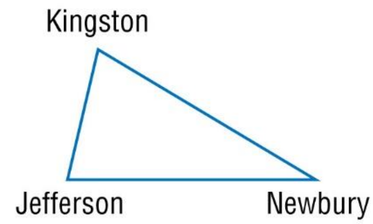
In $\Delta\ PQR, PQ = 7.2$ and $QR = 5.2$. which measure cannot be PR?

a. 7
b. 9
c. 11
d. 13



Example 5-5-4: Proof Using Triangle Inequality Theorem

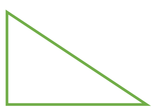
The towns of Jefferson, Kingston, and Newbury are shown in the map below. Prove that driving first from Jefferson to Kingston and then Kingston to Newbury is a greater distance than driving from Jefferson to Newbury.

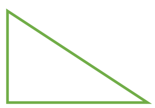


Example 5-5-5: Finding Possible Side Lengths and Angle Measures

a. In ΔABC and ΔDEF, $\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}, AB = 11\ in, ED = 15\ in$, and $m\angle F = 58°$.

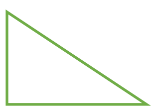
Which of the following is a possible measure for $m\angle C$: 45°, 58°, 80°, or 90°?

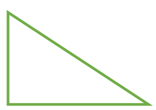




b. In ΔGHI and ΔJKL, $\overline{GH} \cong \overline{JK}, \overline{HI} \cong \overline{KL}, GI = 9cm, m\angle H = 45°$ and $m\angle K = 65°$.

Which of the following is a possible length for $\overline{JL}$: 5cm, 7cm, 9cm or 11cm?





Section Summary

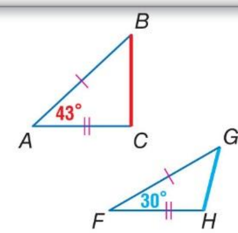
Objectives:

1. Apply the Hinge Theorem or its converse to make comparisons in two triangles.
2. Prove triangle relationships using the Hinge Theorem or its converse.

Theorems Inequalities in Two Triangles

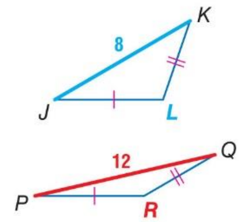
5.13 Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

Example: If $\overline{AB} \cong \overline{FG}$, $\overline{AC} \cong \overline{FH}$, and $m\angle A > m\angle F$, then $BC > GH$.



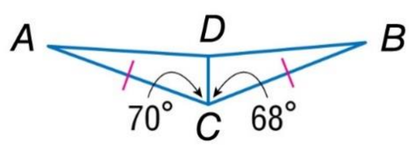
5.14 Converse of the Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle.

Example: If $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, and $PQ > JK$, then $m\angle R > m\angle L$.

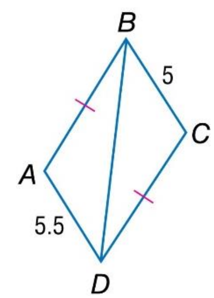


Example 5-6-1: **Use the Hinge Theorem and Its Converse**

A. Compare the measures *AD* and *BD*.



B. Compare the measures $m\angle ABD$ and $m\angle BDC$.

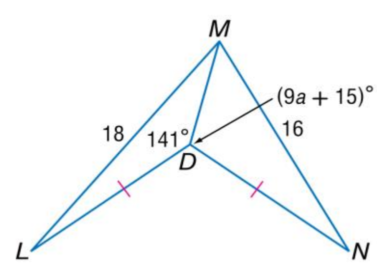


Example 5-6-2: **Use the Hinge Theorem**

Doctors use a straight-leg-raising test to determine the amount of pain felt in a person's back. The patient lies flat on the examining table, and the doctor raises each leg until the patient experiences pain in the back area. Nitan can tolerate the doctor raising his right leg 35° and his left leg 65° from the table. Which leg can Nitan raise higher above the table?

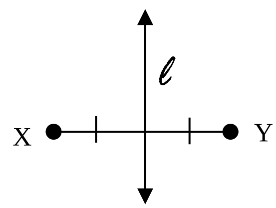
Example 5-6-3: **Apply Algebra to the Relationships in Triangles**

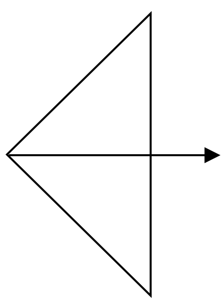
Find the range of possible values for *a*.



Review

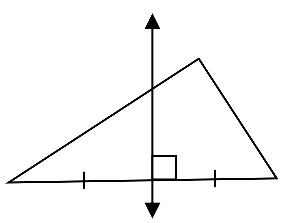
1. I am a segment whose endpoints are a vertex of a triangle and the midpoint of the opposite side. **WHO AM I?**
2. I am the point of concurrency for the angle bisectors of a triangle. **WHO AM I?**
3. I am the point that divides a median into 2/3 and 1/3 parts. **WHO AM I?**
4. I am the segments that intersect to form the point of concurrency called the orthocenter. **WHO AM I?**
5. I am the segment that joins the midpoints of 2 sides of a triangle. **WHO AM I?**
6. I am a segment that extends from a vertex of a triangle to the line containing the opposite side. **WHO AM I?**
7. I am the point of concurrency of the medians. **WHO AM I?**
8. I am the type of proof that starts by assuming the opposite of what is to be proved. **WHO AM I?**
9. I am the line in the diagram **WHO AM I?**





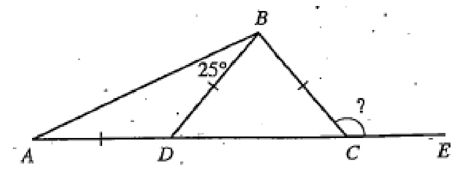
10. I am ray in the diagram. **WHO AM I?**

11. I am a point that is equidistant from the sides of a triangle. **WHO AM I?**
12. I am parallel to the third side of a triangle and ½ the length of the third side. **WHO AM I?**
13. I am equidistant from the vertices of the triangle. **WHO AM I?**
14. I am a point where 3 or more lines intersect. **WHO AM I?**
15. I am the line in the diagram. **WHO AM I?**



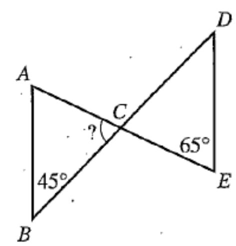
ACT Practice Questions

April 2009: In the figure below, $A, D, C,$ and E are collinear. $\overline{AD}$, $\overline{BD}$, and $\overline{BC}$ are all the same length, and the angle measure of $\angle ABD$ is as marked. What is the degree measure of $\angle BCE$?



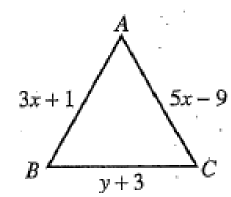
F. 50°
G. 100°
H. 105°
J. 130°
K. 160°

December 2012: In the figure below, C lies on both $\overline{AE}$ and $\overline{BD}$, $\overline{AB}$ and $\overline{DE}$ are parallel and congruent, and 2 angle measures are given. What is the measure of $\angle ACB$?



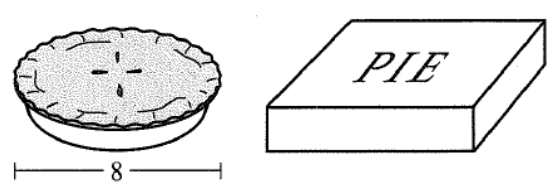
F. 55°
G. 57.5°
H. 65°
J. 67.5°
K. 70°

December 2012: The dimensions of the equilateral triangle ΔABC are given in centimeters in the figure. What is the value of y?



F. 2
G. 5
H. 8
J. 13
K. 16

June 2017: A 3-inch-tall rectangular box with a square base is constructed to hold a circular pie that has a diameter of 8 inches. Both are shown. What is the volume, in cubic inches, of the smallest such box that can hold this pie?



A. 24
B. 64
C. 72
D. 192
E. 512