QUADRILATERALS

Chapter 6

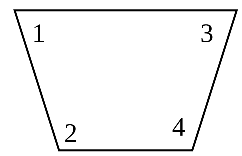
GEOMETRY

Name ________________________________

Hour ___________

6-1 Angles of Polygons

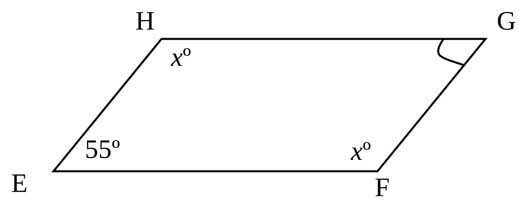
I CAN…	INTERIOR ANGLES OF QUADRILATERALS
Classify polygons abased on their sides and angles find and use the measures of interior and exterior angles of polygons	A ____________________ of a polygon is a segment that joins two _________________ vertices. Like triangles, quadrilaterals have both ____________ and ______________ angles. Interior Angles of a Quadrilateral: The sum of the measures of the interior angles of a quadrilateral is 360°.



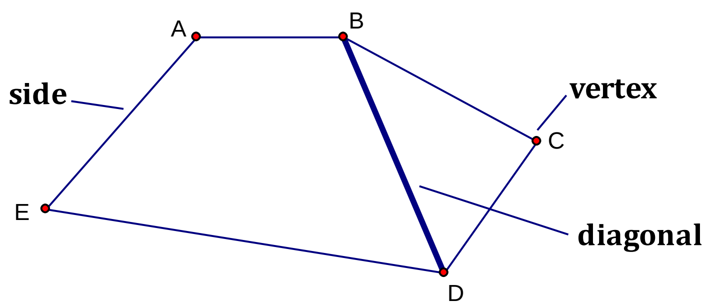
$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

Example 6-1-1: **Interior Angles of a Quadrilateral**

Find $m\angle$F, $m\angle$G, and $m\angle$H.



RECALL



Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n-gon

RECALL POLYGONS

Polygon: A plane ______________ that meets the following conditions.

1. It is formed by ________ or more segments called ____________, such that no _____ sides with a ______________ endpoint are noncollinear.
2. Each side intersects exactly ___________ other sides, one at each _______________.

A polygon is ____________ if no line that contains a side of the polygon contains a point in the interior of the polygon.

A polygon that is not convex is called ___________________ or ________________.

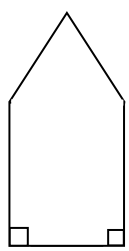
A polygon is ____________________ if all of its sides are congruent.

A polygon is ____________________ if all of its interior angles are congruent.

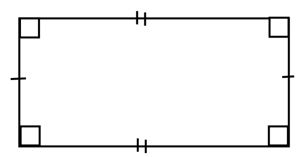
A polygon is _______________ if it is __________________ and ____________________.

Example 6-1-2: Identifying Regular Polygons
Decide whether the polygon is regular.

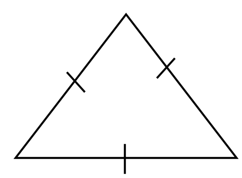
a.



b.



c.

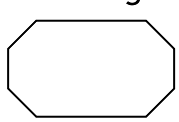


______________ ____________________ _________________

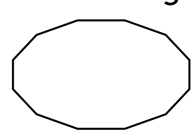
Polygon Interior Angle Sum Theorem	The sum of the interior angle measures of an *n*–sided convex polygon is $(n-2) \cdot 180$.

Example 6-1-3: **Find the <u>sum</u> of the measures of the interior angles of each convex polygon.**

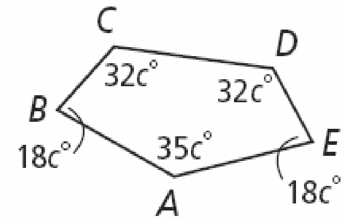
A. octagon n = ___



B. dodecagon n = ___



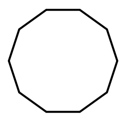
Example 6-1-4: **Find the measure of each interior angle of pentagon ABCDE.**



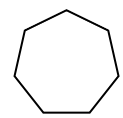
$$\frac{(n-2)180}{n}$$

Example 6-1-5: **Find the measures of <u>each</u> interior angle of each regular polygon.**

A. Decagon n = _____



B. Heptagon n = ____



Find the measure of the exterior angle by subtracting the interior angle from 180.

Exterior total is 360°

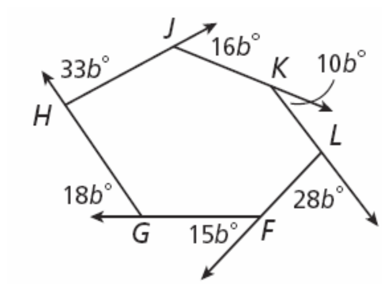
Divide 360 by the exterior angle measure to know how many angles/sides in the polygon.

Example 6-1-6: **The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.**

Polygon Exterior Angle Sum Theorem	The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

Example 6-1-7: **Finding Exterior Angle Measures in Polygons**

A. Find the value of *b* in polygon FGHJKL.



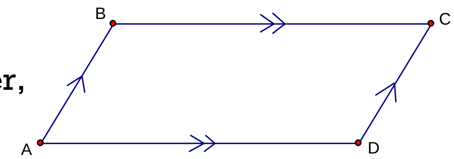
B. Find the measure of each exterior angle of a regular dodecagon.

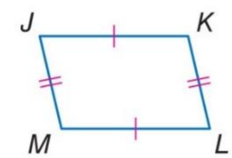
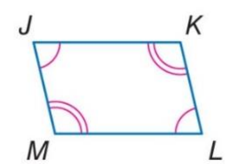
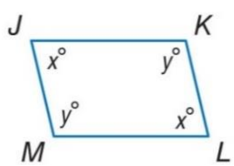
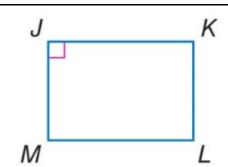
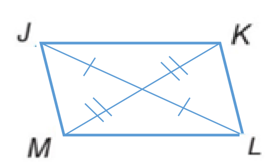
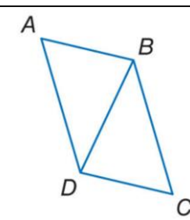
m∠H =_____° m∠J =_____° m∠K =_____°

m∠HL =_____° m∠F =_____° m∠G=_____°

6-2 Parallelograms

I CAN…	PARALLELOGRAMS
prove and apply properties of parallelograms use properties of parallelograms to solve problems	Any polygon with four sides is a ______________________________. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names. Parallelogram → Use the symbol for parallelogram.



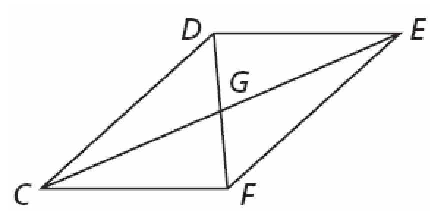
Properties of Parallelograms	
If a quadrilateral is a parallelogram, then … ______________________________ **are congruent.**	
If a quadrilateral is a parallelogram, then … ______________________________ **are congruent.**	
If a quadrilateral is a parallelogram, then … ______________________________ **are supplementary. x + y = 180**	
If a quadrilateral has one right angle, then … **it has ____ right angles.**	
If a quadrilateral is a parallelogram, then … **the __________________ bisect each other.**	
If a quadrilateral is a parallelogram, then … **a ___________ _____________ cuts the parallelogram into 2 triangles.**	

Example 6-2-1: In parallelogram CDEF, $DE = 74\ mm, DG = 31\ mm,$ and $m\angle FCD = 42^\circ$. Find each measure.

A. CF = _____

B. $m\angle EFC$ = _____

C. DF = _____

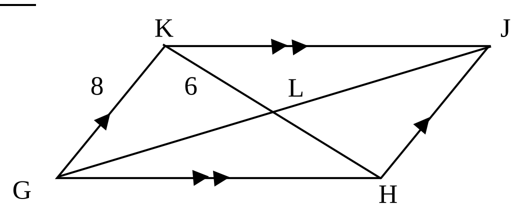


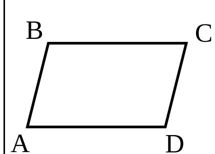
Example 6-2-2: **Using the Properties of Parallelograms**

GHJK is a parallelogram. Find the unknown length.

a. JH = ____

b. LH = ____





Example 6-2-3: **Using the Properties of Parallelograms**

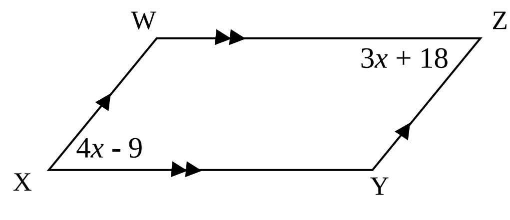
In parallelogram ABCD, m∠C = 105°. Find the angle measure.

a. m∠ A = _____

b. m∠ D= _____

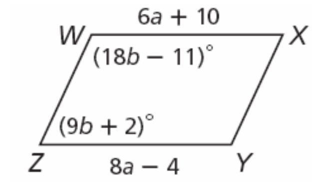
Example 6-2-4: **Using Algebra with Parallelograms**

WXYZ is a parallelogram. Find the value of *x*.



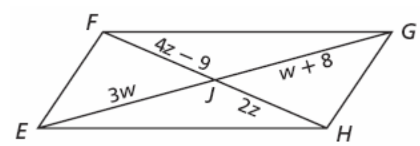
Example 6-2-5: $WXYZ$ **is a parallelogram. Find each measure.**

A. $YZ =$ ______ B. $m\angle Z =$ _______



Example 6-2-6: $EFGH$ **is a parallelogram.**

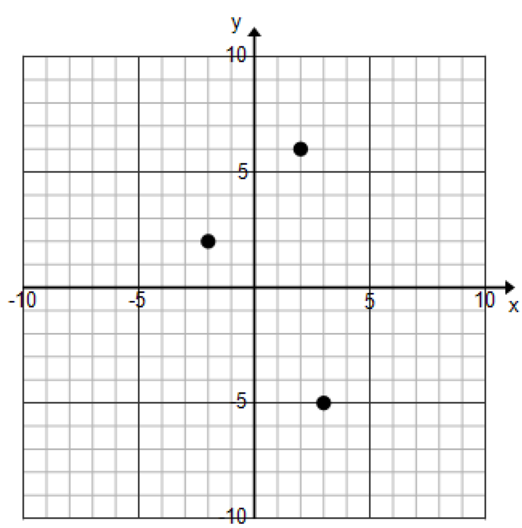
Find each measure.



A. $JG =$ _____ B. $FH =$ _____

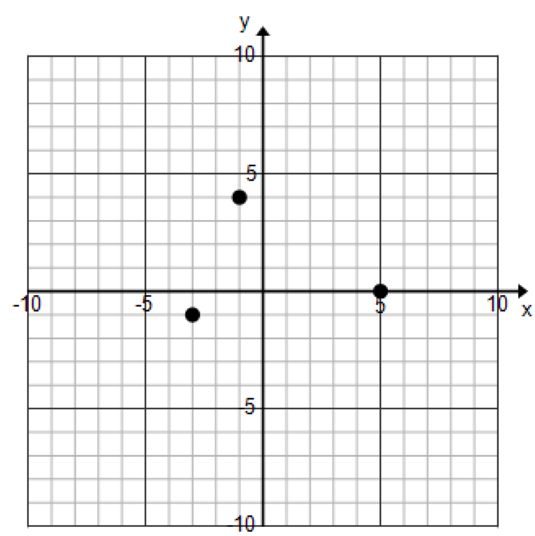
Example 6-2-7: **Parallelograms in the Coordinate Plane**

A. Three vertices of parallelogram JKLM are $J(3,-5), K(-2,2)$, and $L(2,6)$. Find the coordinates of vertex M.



M (,)

B. Three vertices of parallelogram $PQRS$ are $P(-3,-1), Q(-1,4)$, and $S(5,0)$. Find the coordinates of vertex R.

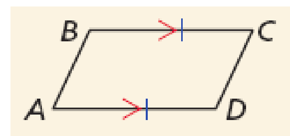
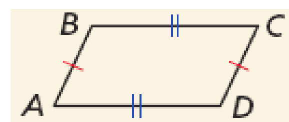
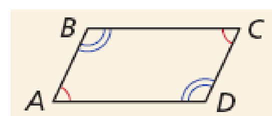
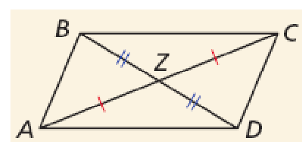


R (,)

6-3 Tests for Parallelograms

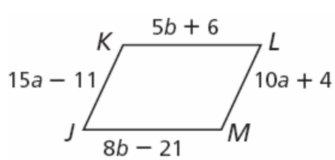
I can…	
Prove that a given quadrilateral is a parallelogram.	You have learned to identify the properties of a parallelogram. NOW you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. You can do this by using the definition of a parallelogram OR the conditions below. **Parallelogram → both pair of ____________________ are parallel.**

Conditions For Parallelograms

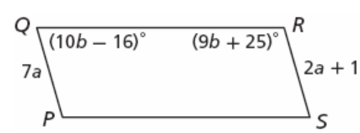
	theorem	example
1	If **opposite sides are** ______________ **and congruent,** then the quadrilateral is a parallelogram.	
2	If ________ __________ **of opposite sides are congruent,** then the quadrilateral is a parallelogram.	
3	If **opposite** ________ **are** ***congruent*** in a quadrilateral, then the quadrilateral is a parallelogram.	
4	If **a quadrilateral's** ______________ **bisect each other,** then the quadrilateral is a parallelogram.	

Example 6-3-1: ***Verifying Figures are Parallelograms***

A. Show that $JKLM$ is a parallelogram for $a = 3$ and $b = 9$

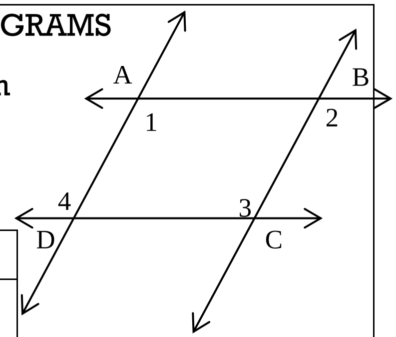


B. Show that $PQRS$ is a parallelogram for $a = 2.4$ and $b = 9$



REASONING ABOUT PARALLELOGRAMS

Example 6-3-2: Proving Facts about Parallelogram

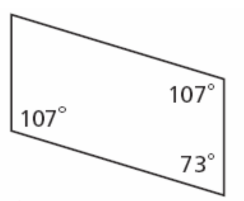


Given: ABCD is a Parallelogram
Prove: $\angle 2 \cong \angle 4$

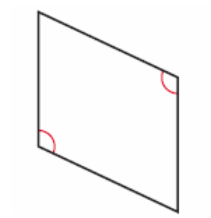
Statements	Reasons
1.	1. Given
2.	2. Def of Parallelogram
3.	3. Corresponding
4.	4. Alt. Interior Angles
5. $\angle 2 \cong \angle 4$	5.

Example 6-3-3: **Applying Conditions for Parallelograms**
Determine if each quadrilateral must be a parallelogram. Justify your answer.

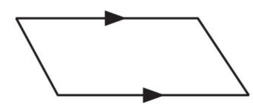
A.



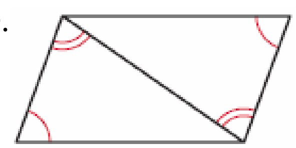
B.



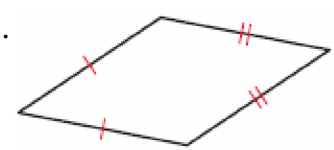
C.



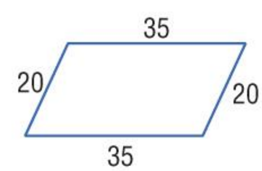
D.



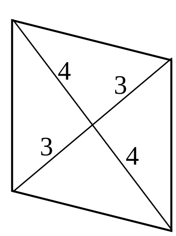
E.



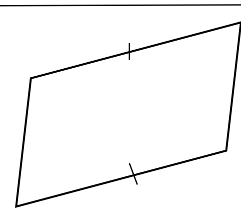
F.



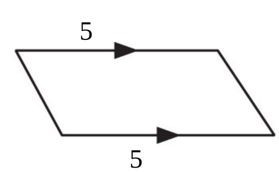
G.



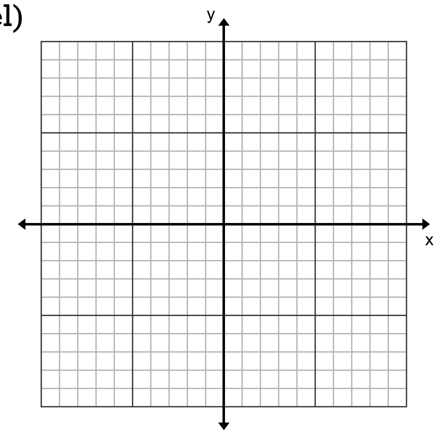
H.



I.

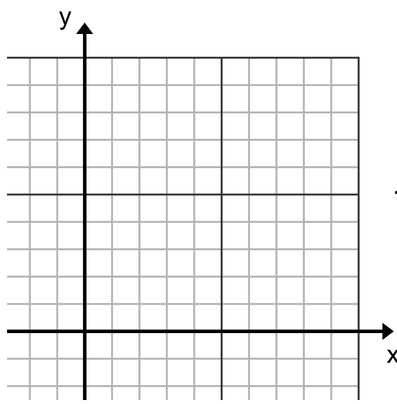


Example 6-3-4: **Proving Parallelograms in the Coordinate Plane –** Show that quadrilateral $JKLM$ is a parallelogram by using the given definition or theorem.(opposite sides parallel)



A. $J(-1,-6), K(-4,-1), L(4,5), M(7,0)$; definition of parallelogram

$\overline{KJ} \overset{?}{\parallel} \overline{ML}$ $\overline{KL} \overset{?}{\parallel} \overline{JM}$?

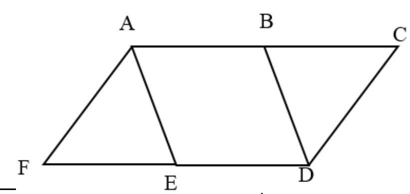


B. $A(2,3), B(6,2), C(5,0), D(1,1)$: quad. with pair of opp. sides ∥ and ≅ → parallelogram

AD $\overset{?}{=}$ BC $\overline{AD} \overset{?}{\parallel} \overline{BC}$

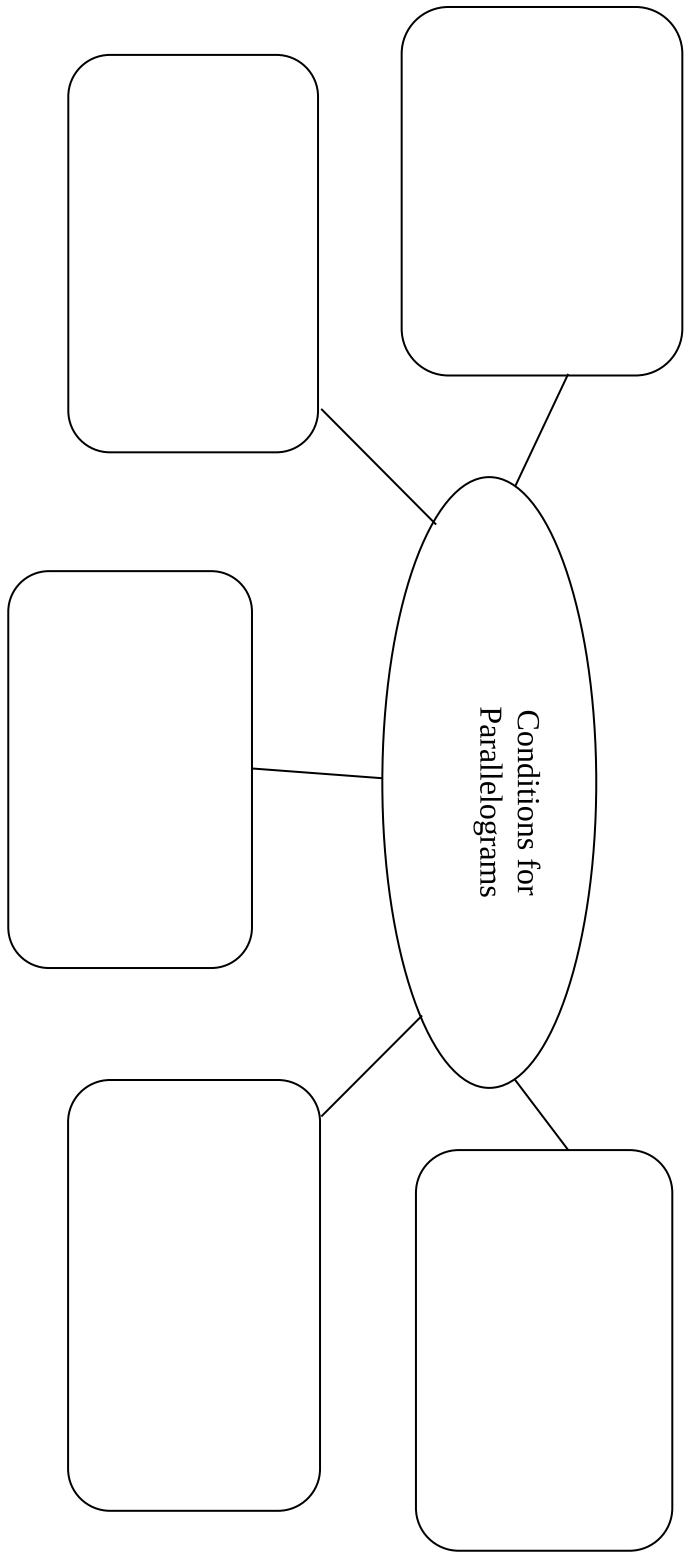
Example 6-3-2: **Proving Facts about Parallelograms**

Given: ACDF is a parallelogram. ABDE is a parallelogram.
Prove: Δ BCD ≅ Δ EFA



Statements	Reasons
1)	1) Given
2)	2) Opposite Sides ≅
3)	3)
4)	4) ≅ angles definition
5) $m\angle FAB = m\angle FAB + m\angle FAB$ $m\angle CDE = m\angle CDB + m\angle BDE$	5)
6)	6) Substitution
7) $m\angle FAB - m\angle EAB = m\angle CBD$ $m\angle FAB - m\angle EAB = m\angle FAE$	7) Subtraction
8)	8)
9)	9) ≅ angles definition
10) Δ BCD ≅ Δ EFA	10)

<u>Section Summary</u> – In each box sketch a parallelogram and label it to show how it meets the conditions for a parallelogram



6-4 Rectangles

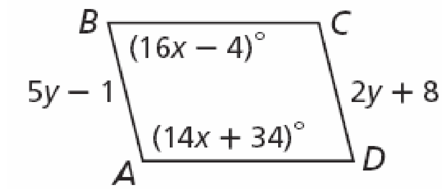
I can …

Prove and apply properties of rectangles.

determine whether parallelograms are rectangles

WARM-UP: $ABCD$ is a parallelogram. Find each measure.

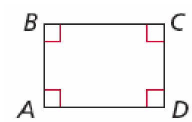
1. $CD =$ _____ 2. $m\angle C =$ _____



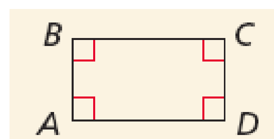
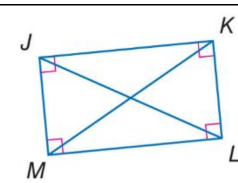
The first type of special quadrilateral we learned about is a

________________________________.

A second type of special quadrilateral is a ______________________________.



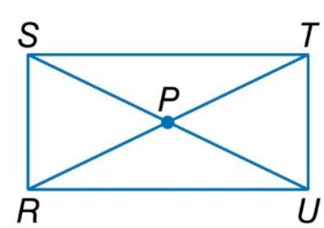
Rectangle *ABCD*

Properties of Rectangles	
Theorem	**Hypothesis**
If a parallelogram is a rectangle, then…	
If a parallelogram is a rectangle, then…	

- Since a rectangle is a special type of a parallelogram, it "inherits" all the properties of parallelograms that you learned in Lesson 6.2.

Example 6-4-1: **Algebra with Rectangles**

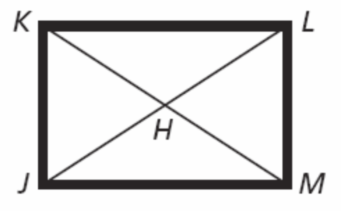
Quadrilateral RSTU is a rectangle. If $m\angle RTU = 8x + 4$ and $m\angle SUR = 3x - 2$, find x.

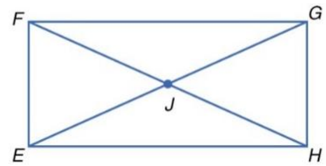


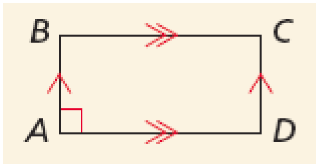
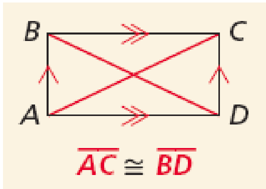
Example 6-4-2: **Using Properties of Rectangles to Find Measures.**

A. $JKLM$ is a rectangular picture frame. $JK = 50\ cm$ and $JL = 86\ cm$.
Find $HM =$ _____ and $LM =$ ____

B. Quadrilateral EFGH is a rectangle. If GH = 6 ft and FH = 15 ft.
Find GJ = ______ and FE = _____



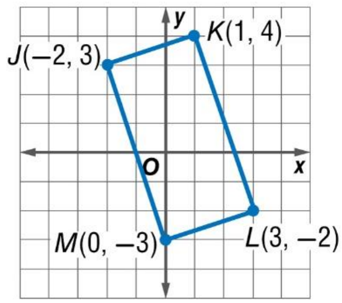


Conditions For Rectangles	
Theorem	**Example**
If then the parallelogram is a rectangle.	
If then the parallelogram is a rectangle.	

Example 6-4-3: **Rectangles in the Coordinate Plane**

Quadrilateral JKLM has vertices J(–2, 3), K(1, 4), L(3, –2), and M(0, –3). Determine whether JKLM is a rectangle using the diagonals. Also verify that the angles are right angles.

Step 1: Verify that it is a parallelogram
Step 2: Verify the lengths of the diagonals are congruent
Step 3: Verify the sides are perpendicular



6-5 Rhombi & Squares

I can….

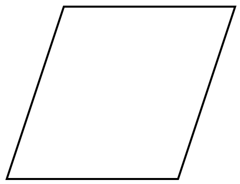
Prove and apply properties of rhombi and squares.

Determine whether parallelograms are rectangles, rhombi, or squares.

RHOMBI AND SQUARES

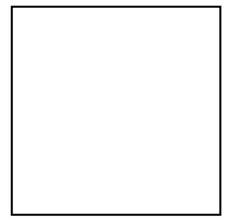
<u>Rhombus</u>

A ___ with _____ congruent sides



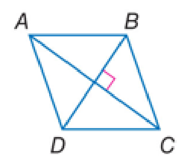
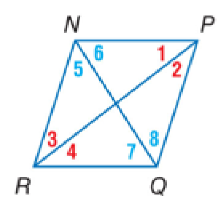
<u>Square</u>

A ___ with _____ congruent sides and ____ right angles.



Rhombus Corollary: A __________ is a rhombus iff it has four ___________ __________.

Square Corollary: A __________ is a square iff it is a _________ and a ____________.

Properties of rhombi	
THEOREM	
If a parallelogram is a rhombus, then…	
If a parallelogram is a rhombus, then…	

- Like a rectangle, a rhombus is a parallelogram. So you can apply the properties of parallelograms to rhombi.
- Therefore, a square has the **<u>properties of all three</u>**.

Example 6-5-1: **Describing a Special Parallelogram**

Decide whether the statement is *always*, *sometimes*, or *never* true.

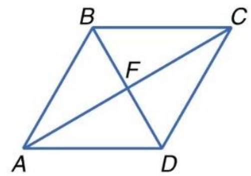
a. A rectangle is a square. b. A square is a rhombus.

Example 6-5-2: **Using properties of squares**.
$QRST$ is a square. What else do you know about $QRST$?
-Diagonals are ____ -4 sides are ____ -Diagonals bisect ___________

-Opposite sides _____ -4 right _______ -Diagonals are _____

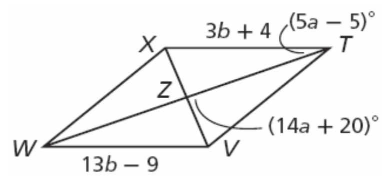
Example 6-5-3: **Using properties of rhombi**.

A. $ABCD$ is a rhombus. If $m\angle ABC = 126°$, find $m\angle CDB$. Also find $m\angle CFD$.



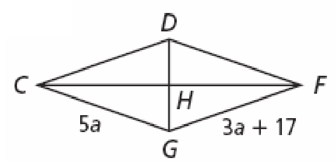
$m\angle CDB =$ ____________ $m\angle CFD =$ ____________

B. $TVWX$ is a rhombus. Find TV and $m\angle VTZ$.



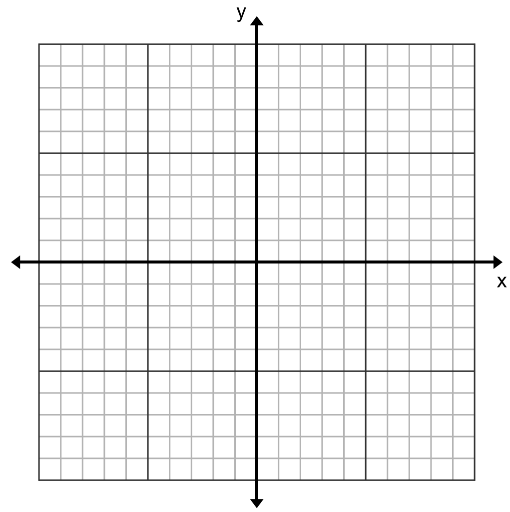
$TV =$ ____________ $m\angle VTZ =$ ____________

C. $CDFG$ is a rhombus. Find CD and $m\angle GCH$ if $m\angle GCD = (b + 3)^{\circ}$ and $m\angle CDF = (6b - 40)^{\circ}$.



$CD =$ ____________ $m\angle GCH =$ ____________

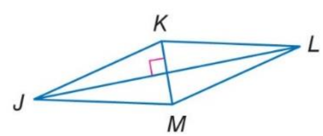
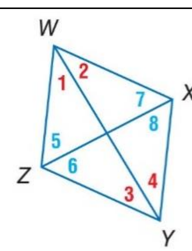
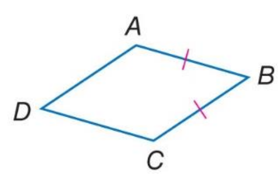
Example 6-5-4: **Verifying Properties of Squares.**
The vertices of square $STVW$ are $S(-5,-4), T(0,2), V(6,-3)$, and $W(1,-9)$. Show that the diagonals of square $STVW$ are congruent perpendicular bisectors of each other.



Step 1: Show that $\overline{SV}$ and $\overline{TW}$ are congruent.

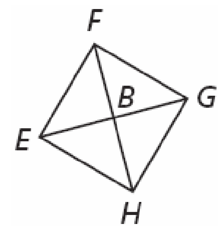
Step 2: Show that $\overline{SV}$ and $\overline{TW}$ are perpendicular.

Step 3: Show that $\overline{SV}$ and $\overline{TW}$ bisect each other. (Use midpoint formula.)

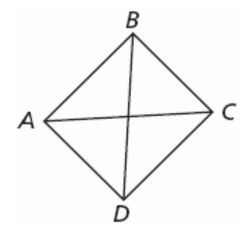
Conditions for Rhombi	
Theorem	**Example**
Then a parallelogram is a Rhombus	
Then a parallelogram is a Rhombus	
Then a parallelogram is a Rhombus	

Example 6-5-5: **Applying Conditions for Special Parallelograms**
Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

A. Given: $\overline{EF} \cong \overline{FG}, \overline{EG} \perp \overline{FH}$
Conclusion: *EFGH* is a rhombus.

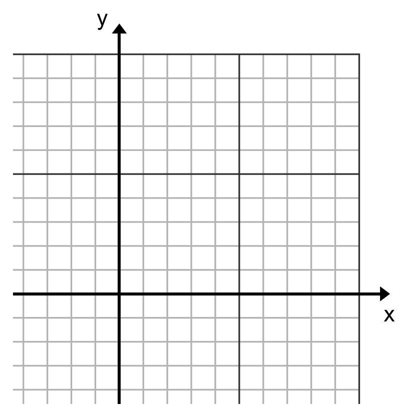


B. Given: $\angle ABC$ is a right angle.
Conclusion: *ABCD* is a rectangle.

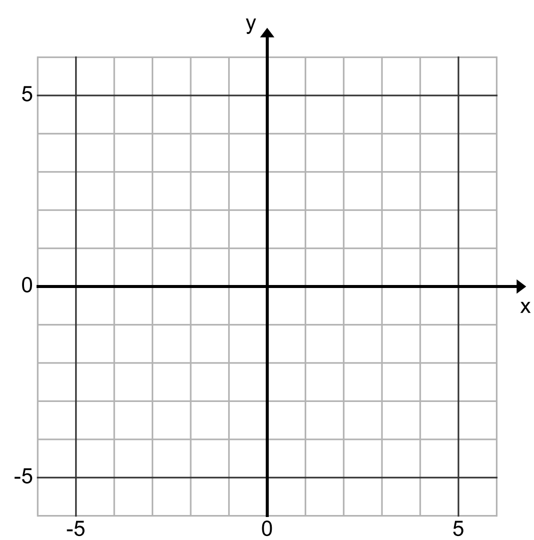


Example 6-5-6: **Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**
(Steps: Graph parallelogram; determine if rectangle, then rhombus, then square.)

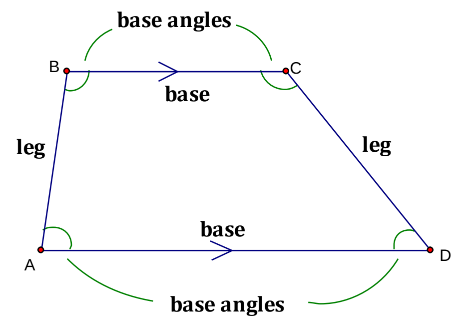
A. $P(-1, 4), Q(2, 6), R(4, 3), S(1, 1)$

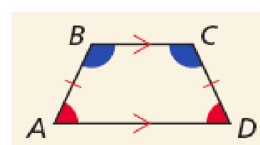
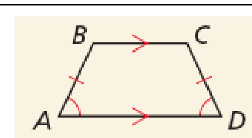
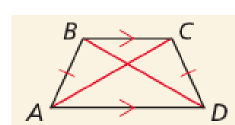


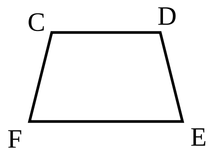
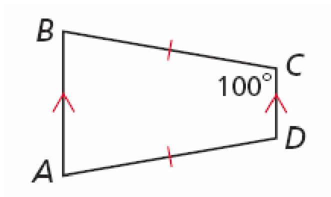
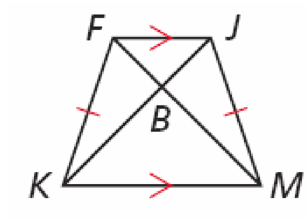
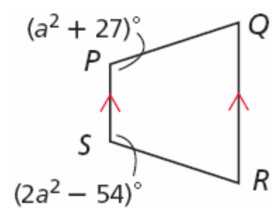
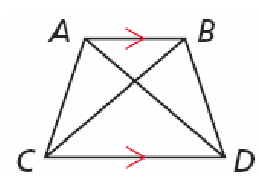
B. $W(0, 1), X(4, 2), Y(3, -2), Z(-1, -3)$



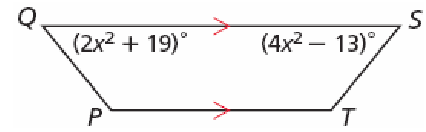
6-6 Trapezoids

I can…	**PROPERTIES OF TRAPEZOIDS**
	A ______________ is a quadrilateral with ___________ ________ pair of __________ sides. The __________ sides of a trapezoid are the ____________ of the trapezoid. For each of the bases of a trapezoid, there is a pair of ______ ________, which are the two __________ that have that base as a side. The ____-________ sides of a trapezoid are called __________. If the _______ of a trapezoid are congruent, then the trapezoid is an __________ _________. The ___________ of a trapezoid is the segment that connects the midpoints of its legs. 

Isosceles Trapezoids	
Theorem	**Hypothesis**
If a trapezoid is isosceles, then…	
If a trapezoid has , then…	
A trapezoid is isosceles iff it is	

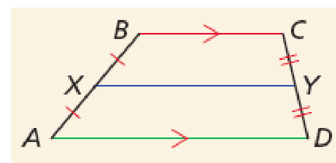
	Example 6-6-4: **Using Properties of Isosceles Trapezoids** *CDEF* is an isosceles trapezoid with CE = 10 and $m\angle E = 95°$. Find DF, $m\angle C$, $m\angle D$, and $m\angle F$. Example 6-6-5: **Using Properties of Isosceles Trapezoids.** A. Find $m\angle A =$ ____  B. $KB = 21.9$ and $MF = 32.7$. Find *FB* =____  Example 6-6-6: **Applying Conditions for Isosceles Trapezoids.** A. Find the value of a so that $PQRS$ is isosceles. 
	B. $AD = 12x - 11$ and $BC = 9x - 2$. Find the value of x so that $ABCD$ is isosceles. 

C. Find the value of x so that $PQST$ is isosceles.



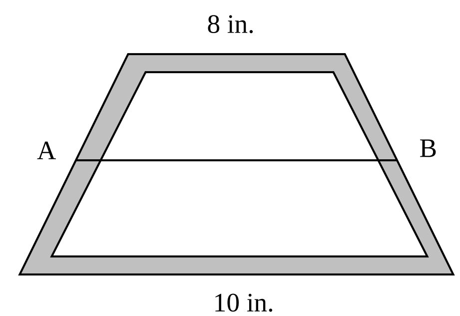
Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base and its length is one half of the sum of the lengths of the bases.



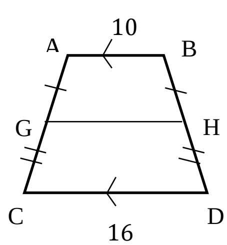
Example 6-6-8: **Finding Midsegment Lengths of Trapezoids**

A potter crafts a trapezoidal relish dish, placing a divider, shown by $\overline{AB}$, in the middle of the dish. How long must the divider be to ensure that it divides the legs in half?

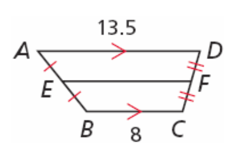


Example 6-6-9: **Finding Lengths Using Midsegments**

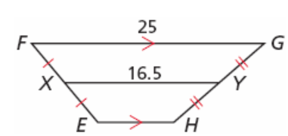
A. Find *GH*.



B. Find *EF*.



C. Find *EH*.



SPECIAL QUADRILATERALS - REVIEW

<table>
<tr><td>I can…</td><td>Summarizing Properties of Quadrilaterals</td></tr>
<tr><td></td><td>quadrilateral

kite parallelogram trapezoid

rhombus rectangle

square

Example 1: Identifying Quadrilaterals

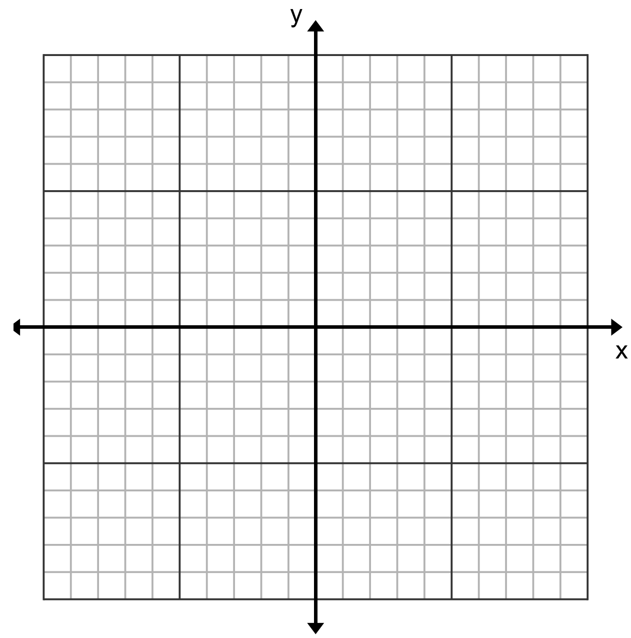
ABCD has at least two congruent consecutive sides. What quadrilaterals meet this condition?</td></tr>
<tr><td></td><td>Example 2: Connecting Midpoints of Sides
When you join the midpoints of the sides of an isosceles trapezoid in order, what special quadrilateral is formed why?</td></tr>
</table>

Three ways to prove a quadrilateral is a rhombus

1. You can use the definition and show that the quadrilateral is a *parallelogram* that has four congruent sides. It is easier, however, to use the Rhombus corollary and simplify show that all four sides of the quadrilateral are congruent.
2. Show that the quadrilateral is a parallelogram *and* that the diagonals are perpendicular.
3. Show that the quadrilateral is a parallelogram *and* that each diagonal bisects a pair of opposite angles.

Example 3: Proving a Quadrilateral is a Rhombus

The coordinates of *ABCD* are A(-2, 5), B (1, 8), C (4, 5), and D (1, 2). Show that *ABCD* is a Rhombus.

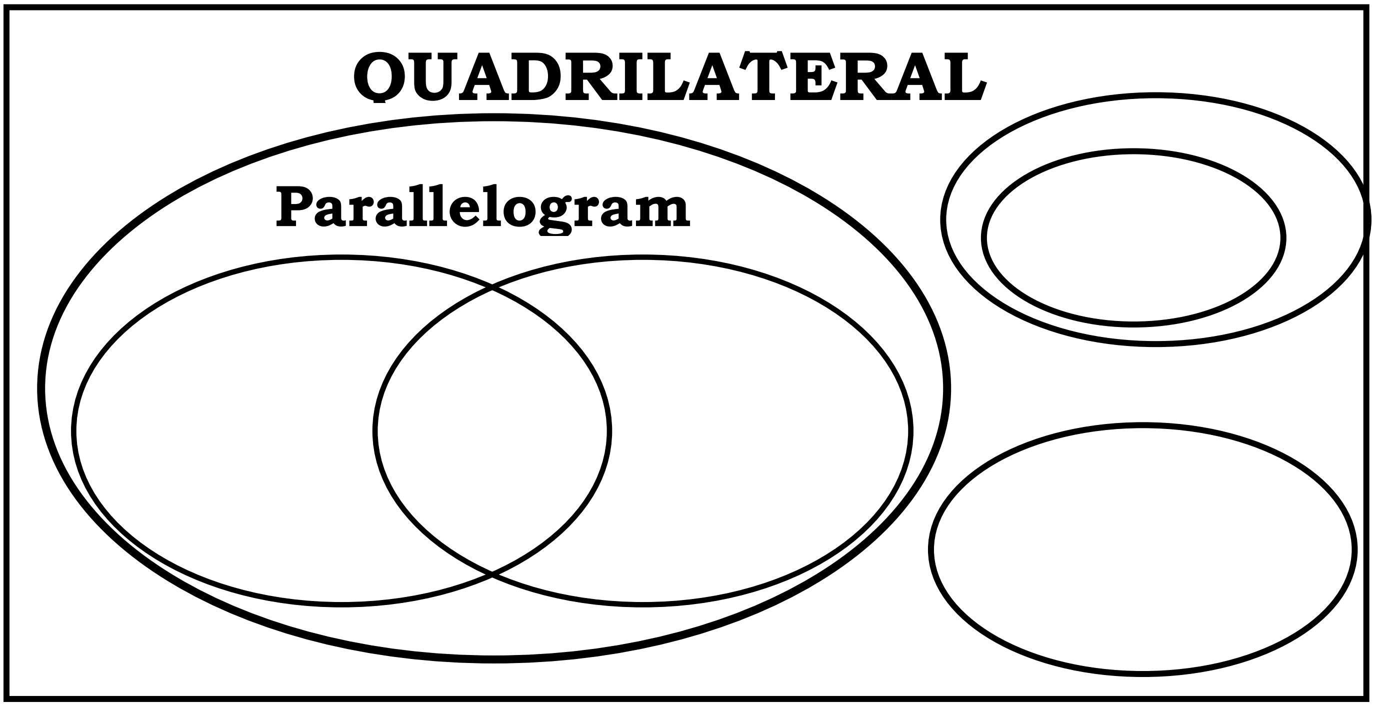


Parallelogram Check:
Diagonals bisect each other?
AC = BD

Rhombus Check:
Diagonals Perpendicular?
$\overline{AC} \perp \overline{BD}$

Example 5: Identifying a Quadrilateral

The diagonals of *ABCD* intersect at point *N* to form four congruent isosceles triangles: $\Delta ANB \cong \Delta CNB \cong \Delta CND \cong \Delta AND$. What type of quadrilateral is *ABCD*? Prove that your answer is correct.


QUADRILATERAL
Parallelogram