CHAPTER 7

Proportions and Similarity
Geometry

Name ______________________________ Hr ______

7-1 Ratio and Proportion

I can…

Write and simplify ratios.

Use proportions to solve problems

RATIO compares two numbers by division.

Written 3 ways: a to b, a:b, or ____, where $b \neq 0$.

Example 7-1-1: **Writing Ratios**

The number of students who participate in sports programs at Central High School is 520. The total number of students in the school is 1850. Find the athlete-to-student ratio to the nearest tenth.

Example 7-1-2: **Writing and Simplifying Ratios**

a) The ratio of the side lengths of a triangle is 4:7:5, and its perimeter is 96 cm. What is the length of the shortest side?

b) In ΔEFG, the ratio of the measures of the angles is 5:12:13. Find the measures of the angles.

Proportion

In the proportion $\frac{a}{b} = \frac{c}{d}$

a and d are the ____________________

b and c are the ____________________

KeyConcept Cross Products Property

Words	In a proportion, the product of the extremes equals the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ when $b \neq 0$ and $d \neq 0$, then $ad = bc$.
Example	If $\frac{4}{10} = \frac{6}{15}$, then $4 \cdot 15 = 10 \cdot 6$.

Example 7-1-3: Solving Proportions

A. $\frac{7}{y} = \frac{56}{72}$

B. $\frac{4x-5}{3} = \frac{-26}{6}$

Example 7-1-4: Using Properties of Proportions

Given that $18c = 24d$, find the ratio of d to c in simplest form.

Example 7-1-5: Real World Application

Monique randomly surveyed 30 students from her class and found that 18 had a dog or a cat for a pet. If there are 870 students in Monique's school, predict the total number of students with a dog or a cat.

KeyConcept Equivalent Proportions

Symbols The following proportions are equivalent.

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{b}{a} = \frac{d}{c}, \quad \frac{a}{c} = \frac{b}{d}, \quad \frac{c}{a} = \frac{d}{b}$$

Examples

$$\frac{28}{50} = \frac{x}{755}, \frac{50}{28} = \frac{755}{x}, \frac{28}{x} = \frac{50}{755}, \frac{x}{28} = \frac{755}{50}.$$

7-2 Ratios in Similar Polygons

I can…

Solve problems using the properties of similar polygons.

Use proportions to identify similar polygons.

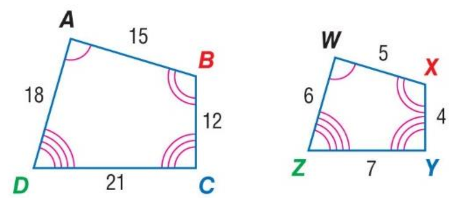
Figures that are **similar** (~) have the same shape but not necessarily the same size.

Two polygons are _____________ ____________ iff their corresponding angles are _______________ and their corresponding sides are ______________________.

KeyConcept Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

Example In the diagram below, *ABCD* is similar to *WXYZ*.



Corresponding angles

$\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle D \cong \angle Z$

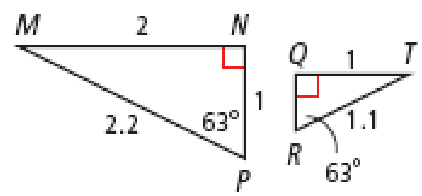
Corresponding sides

$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$

Symbols $ABCD \sim WXYZ$

<u>Example 7-2-1: Describing Similar Polygons</u>

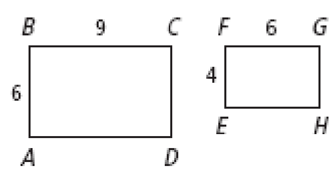
Identify the pairs of congruent angles and corresponding sides.



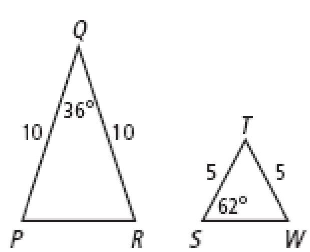
<u>Example 7-2-2 – Identifying Similar Polygons</u>

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

A.

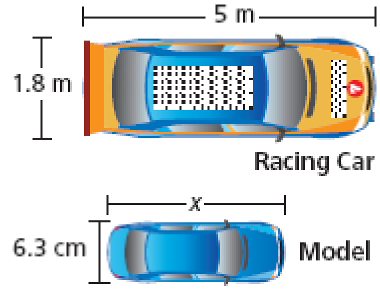


B.



<u>Example 7-2-3 – Real World Application</u>

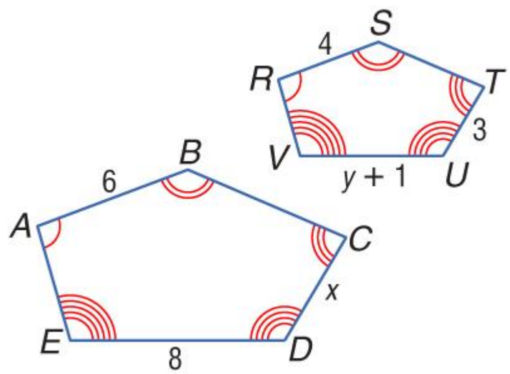
Find the length of the model to the nearest tenth of a centimeter.



<u>Example 7-2-4: Use Similar Figures to Find Missing Measures</u>

a) The two polygons are similar. Find *x.*

b) The two polygons are similar. Find *y.*

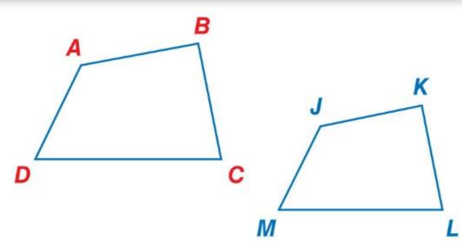


Theorem 7.1 Perimeters of Similar Polygons

If two polygons are similar, then their perimeters are proportional to the scale factor between them.

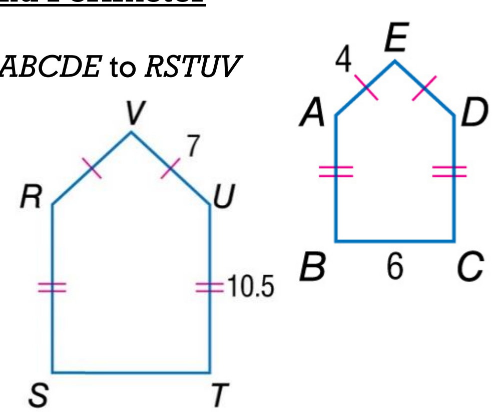
Example If *ABCD* ~ *JKLM*, then

$$\frac{AB + BC + CD + DA}{JK + KL + LM + MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}.$$



<u>Example 7-2-5: Use a Scale Factor to Find Perimeter</u>

If *ABCDE* ~ *RSTUV*, find the scale factor of *ABCDE* to *RSTUV* and the perimeter of each polygon.



7-3 Similar Triangles

I can…

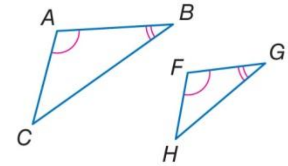
Use congruence and similarity criteria for triangles to solve problems.

Use congruence and similarity criteria for triangles to prove relationships in geometric figures.

Postulate 7.1 Angle-Angle (AA) Similarity

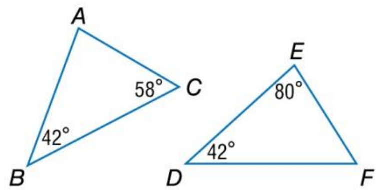
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.

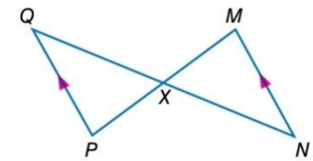


Example 7-3-1: Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

A.



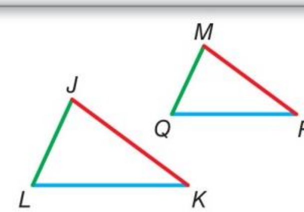
B.



Theorems Points on Perpendicular Bisectors

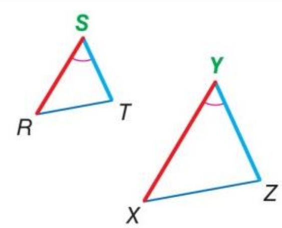
7.2 Side-Side-Side (SSS) Similarity
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Example If $\frac{JK}{MP} = \frac{KL}{PQ} = \frac{LJ}{QM}$, then $\triangle JKL \sim \triangle MPQ$.



7.3 Side-Angle-Side (SAS) Similarity
If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

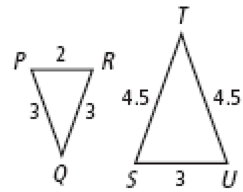
Example If $\frac{RS}{XY} = \frac{ST}{YZ}$ and $\angle S \cong \angle Y$, then $\triangle RST \sim \triangle XYZ$.



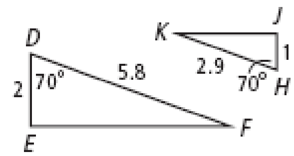
Example 7-3-2: **Verifying Triangle Similarity**

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

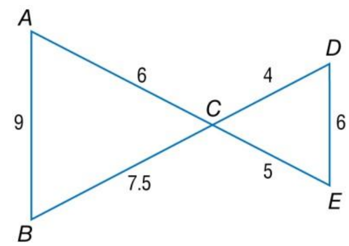
A. $\triangle PQR$ and $\triangle STU$



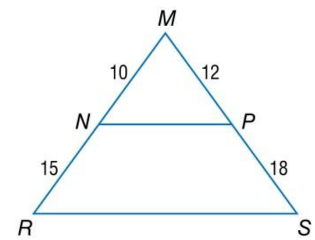
B. $\triangle DEF$ and $\triangle HJK$



C. $\triangle ABC$ and $\triangle EDC$

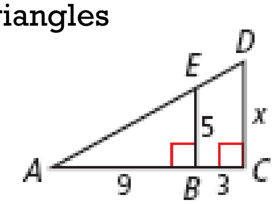


D. $\triangle MNP$ and $\triangle MRS$

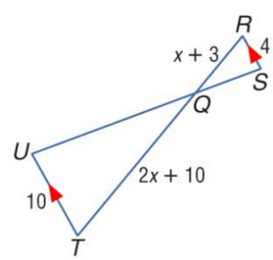


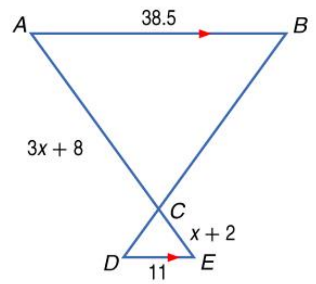
Example 7-3-3: Finding Lengths in Similar Triangles

Explain why $\triangle ABE \sim \triangle ACD$, and then find CD.



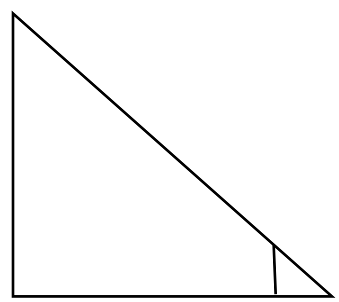
Example 7-3-4: Given $\overline{RS} \parallel \overline{UT}$, RS = 4, RQ = x + 3, QT = 2x + 10, UT = 10, find RQ and QT.



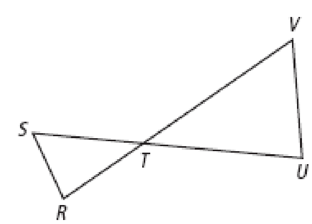


Example 7-3-5: Given $\overline{AB} \parallel \overline{DE}$, AB = 38.5, DE = 11, AC = 3x + 8, and CE = x + 2, find AC.

Example 7-3-6: Josh wanted to measure the height of the Sears Tower in Chicago. He used a 12-foot light pole and measured its shadow at 1 p.m. The length of the shadow was 2 feet. Then he measured the length of Sears Tower's shadow and it was 242 feet at the same time. What is the height of the Sears Tower? (Make a sketch of the situation.)



Example 7-3-7: Writing Proofs with Similar Triangles



Given: $3UT = 5RT$ and $3VT = 5ST$

Prove: $\Delta UVT \sim \Delta RST$

Statements	Reasons
1. $3UT = 5RT$ and $3VT = 5ST$	1.
2. $\frac{UT}{RT} = \frac{5}{3} = \frac{VT}{ST}$	2.
3. $\angle STR \cong \angle UTV$	3.
4. $\Delta UVT \sim \Delta RST$	4.

Example 7-3-9: Real World Application

Find the length of $\overline{BC}$ to the nearest tenth.

$\frac{BF}{17+BF} = \frac{6.5}{24}$

$24BF = 110.5 + 6.5BF$

$17.5BF = 110.5$

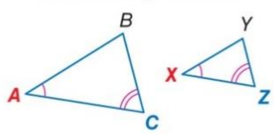
$BF = 6.314\sim$ 6.3 ft

Theorem 7.4 Properties of Similarity

Reflexive Property of Similarity	$\triangle ABC \sim \triangle ABC$
Symmetric Property of Similarity	If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.
Transitive Property of Similarity	If $\triangle ABC \sim \triangle DEF$, and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.

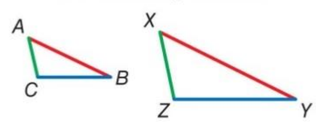
ConceptSummary Triangle Similarity

AA Similarity Postulate



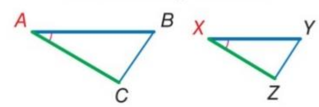
If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$.

SSS Similarity Theorem



If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

SAS Similarity Theorem



If $\angle A \cong \angle X$ and $\frac{AB}{XY} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

Think and Discuss

	Congruence	Similarity
SSS		
SAS		
AA		

7-4 Applying Properties of Similar Triangles

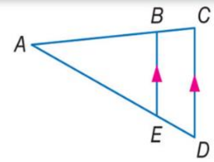
I can…

Use proportional parts within triangles.

Use proportional parts with parallel lines.

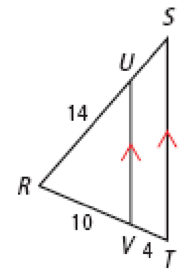
Theorem 7.5 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



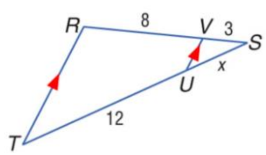
Example If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$.

Example 7-4-1: **Using Proportionality Theorem**
Find US.

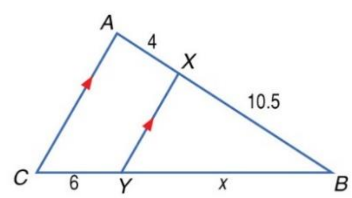


Example 7-4-2: **Finding the Length of a Segment**

A. In ΔRST, $\overline{RT} \parallel \overline{VU}$, $SV = 3$, $VR = 8$, and $UT = 12$. Find SU.

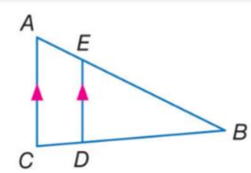


B. In ΔABC, $\overline{AC} \parallel \overline{XY}$, $AX = 4$, $XB = 10.5$, and $CY = 6$. Find BY.



Theorem 7.6 Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

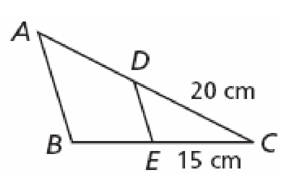


Example If $\frac{AE}{EB} = \frac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.

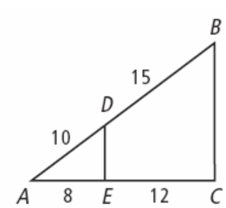
Verify by Converse Δ Proportion Thm

Example 7-4-3: **Using Proportionality Theorem Converse to Verify Segments are parallel**

A. AC = 36 cm and BC = 27 cm. Verify that $\overline{DE} \parallel \overline{AB}$.



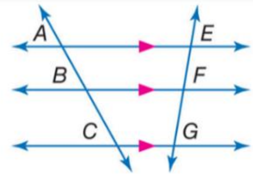
B. Verify that $\overline{DE} \parallel \overline{BC}$.



Corollary 7.1 Proportional Parts of Parallel Lines

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

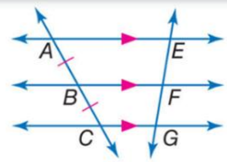
Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.



Corollary 7.2 Congruent Parts of Parallel Lines

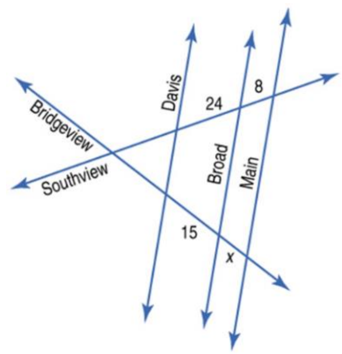
If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.



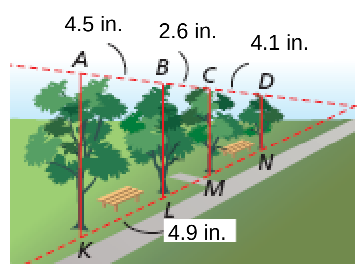
Example 7-4-4: **Using Proportional Segments of Transversals**

In the figure, Davis, Broad, and Main Streets are all parallel. The figure shows the distances in between city blocks. Find *x*.



Example 7-4-5: **Using Proportionality Theorems**

Suppose that the artist decided to make a larger sketch of the trees. In the figure, if *AB* = 4.5 in., *BC* = 2.6 in., *CD* = 4.1 in., and *KL* = 4.9 in., find *LM* and *MN* to the nearest tenth of an inch.



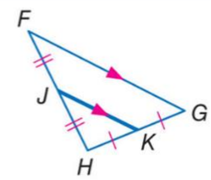
MIDSEGMENT THEOREM

A _______________ of a triangle is a segment that connects the _____________ of two sides of a triangle.

Theorem 7.7 Triangle Midsegment Theorem

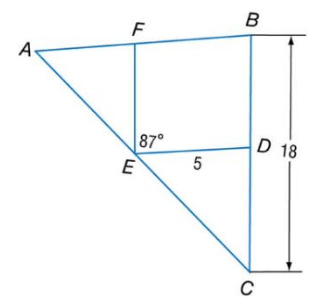
A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.



Example 7-4-6: **Using the Triangle Midsegment Theorem**

In the figure, DE and EF are midsegments of ΔABC. Find AB, FE, and $m\angle AFE$.



A. $AB =$ B. $FE =$

C. $m\angle AFE =$

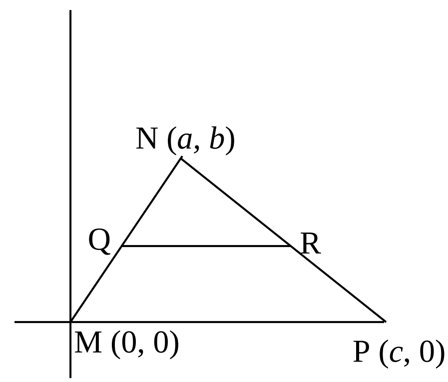
Example 7-4-7: **Midsegment in the Coordinate Plane**

a. What are coordinates of Q? of R?

Q (,) R (,)

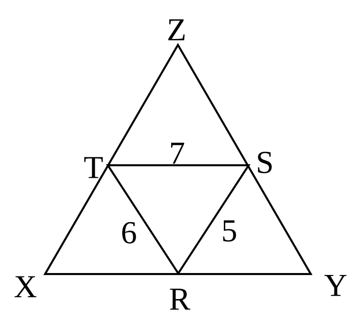
b. Why is $\overline{QR} \parallel \overline{MP}$?

QR Slope = ___ MP Slope = ___



Example 7-4-8: **Midsegment and Perimeter**

$\overline{RS}, \overline{ST}$ and $\overline{RT}$ are midsegments in $\Delta\, XYZ$. Find the perimeter of $\Delta\, XYZ$.



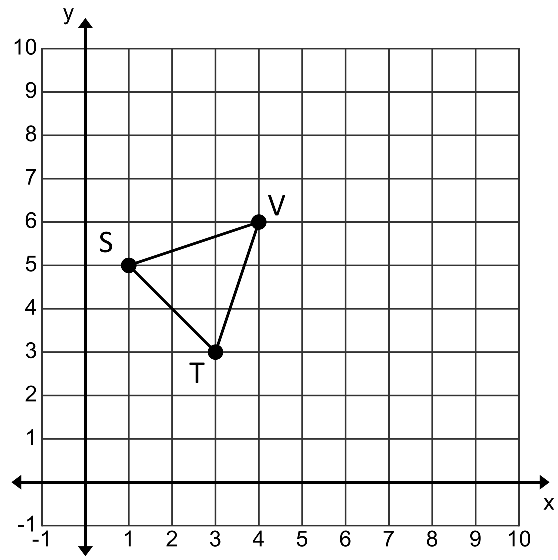
Example 7-4-9: **Midsegment and Midpoints**

The <u>midpoints</u> of the sides of a triangle are S (1, 5), T (3, 3) and V (4, 6). What are the coordinates of the vertices of the triangle?

VT slope = ____

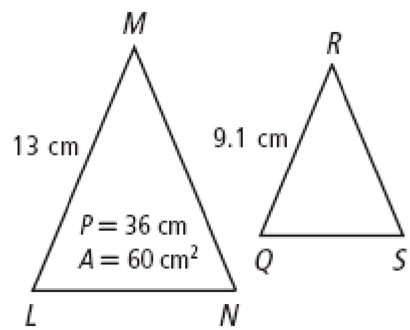
ST slope = ____

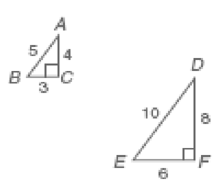
SV slope = ____



Example 7-4-10: **Using Ratios to Find Perimeters and Areas**

Given that $\triangle LMN \sim \triangle QRS$, find the perimeter and area of $\triangle QRS$.



STATEMENT	RATIO
$\triangle ABC \sim \triangle DEF$ 	Similarity ratio: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$ Perimeter ratio: $\frac{\text{perimeter } \triangle ABC}{\text{perimeter } \triangle DEF} = \frac{12}{24} = \frac{1}{2}$ Area ratio: $\frac{\text{area } \triangle ABC}{\text{area } \triangle DEF} = \frac{6}{24} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

7.6 & 9.6 Dilations and Similarity

I can…	
Identify similarity transformations. Verify similarity after a similarity transformation.	

Dilation is an _____________ *or a* ________________.

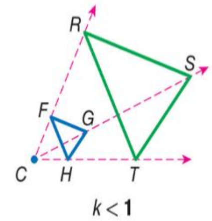
Scale Factor

ConceptSummary Types of Dilations

A dilation with a scale factor greater than 1 produces an **enlargement**, or an image that is larger than the original figure.

Symbols If $k > 1$, the dilation is an enlargement.

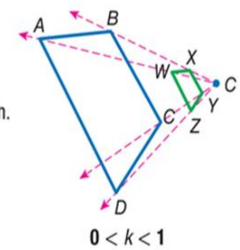
Example $\triangle FGH$ is dilated by a scale factor of 3 to produce $\triangle RST$. Since $3 > 1$, $\triangle RST$ is an enlargement of $\triangle FGH$.



A dilation with a scale factor between 0 and 1 produces a **reduction**, an image that is smaller than the original figure.

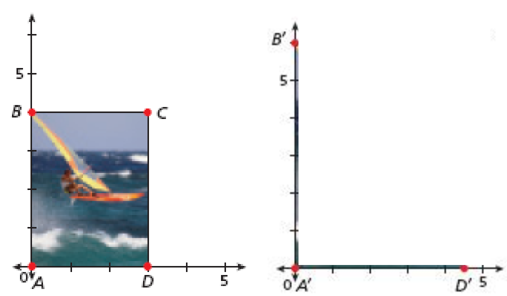
Symbols If $0 < k < 1$, the dilation is a reduction.

Example $ABCD$ is dilated by a scale factor of $\frac{1}{4}$ to produce $WXYZ$. Since $0 < \frac{1}{4} < 1$, $WXYZ$ is a reduction of $ABCD$.



Example 7-6-1: **Computer Graphic Application**

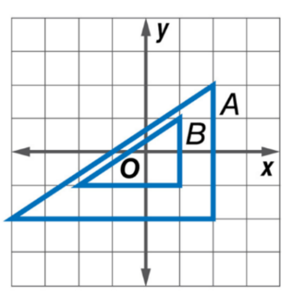
Draw the border of the photo after dilation with scale factor $\frac{5}{2}$.



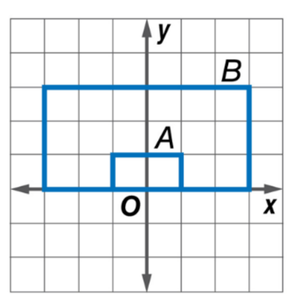
Example 7-6-2: **Identify the Dilation and find its Scale Factor**

Determine whether the dilation from Figure *A* to Figure *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

A.



B.



A to B

Enlargement or reduction

Scale factor:_________

A to B

Enlargement or reduction

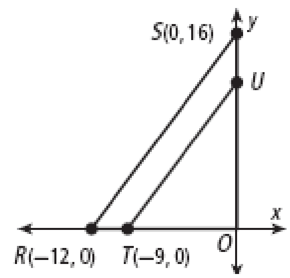
Scale factor:_________

Example 7-6-3: **Find and Use a Scale Factor**
PHOTOCOPYING A photocopy of a receipt is 1.5 inches wide and 4 inches long. By what percent should the receipt be enlarged so that its image is 2 times the original? What will be the dimensions of the enlarged image?

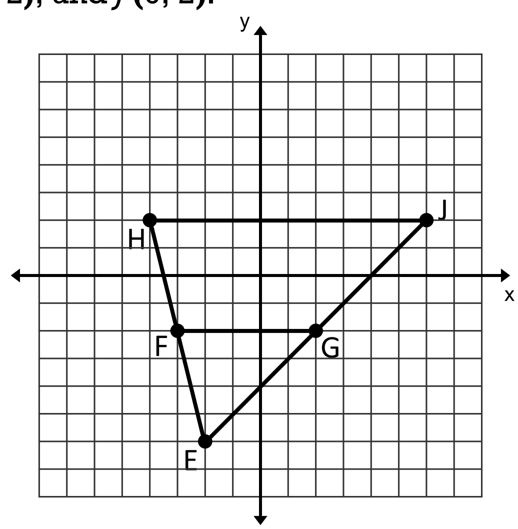
Note: You can verify that a dilation produces a similar figure by comparing corresponding sides and angles. For triangles, you can also use AA~, SSS~, or SAS~.

Example 7-6-4: **Finding Coordinates of Similar Triangles**

Given that $\Delta TUO \sim \Delta RSO$, find the coordinates of *U* and the scale factor.



Example 7-6-5: **Proving Triangles are Similar**
Given: *E* **(**-2, -6**)**, *F* **(**-3, -2**)**, *G* **(**2, -2**)**, *H* **(**-4, 2**)**, and *J* **(**6, 2**)**.
Prove: $\Delta EHJ \sim \Delta EFG$

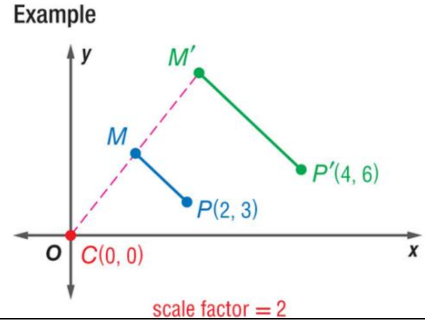


KeyConcept Dilations in the Coordinate Plane

Words To find the coordinates of an image after a dilation centered at the origin, multiply the *x*- and *y*-coordinates of each point on the preimage by the scale factor of the dilation, *k*.

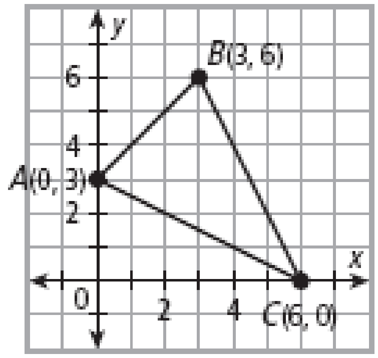
Symbols $(x, y) \rightarrow (kx, ky)$

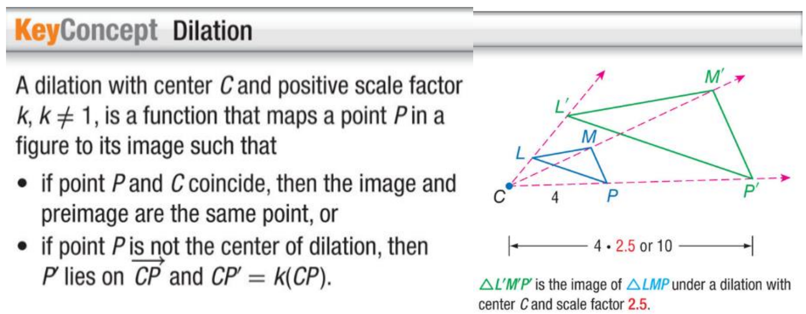
Example



Example 7-6-6: **Using the SSS Similarity Theorem**

Graph the image of ΔABC after a dilation with scale factor $\frac{2}{3}$.





KeyConcept Dilation

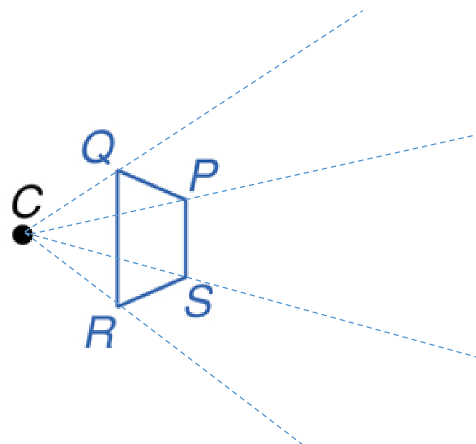
A dilation with center C and positive scale factor k, $k \neq 1$, is a function that maps a point P in a figure to its image such that

- if point P and C coincide, then the image and preimage are the same point, or
- if point P is not the center of dilation, then P' lies on $\overrightarrow{CP}$ and $CP' = k(CP)$.

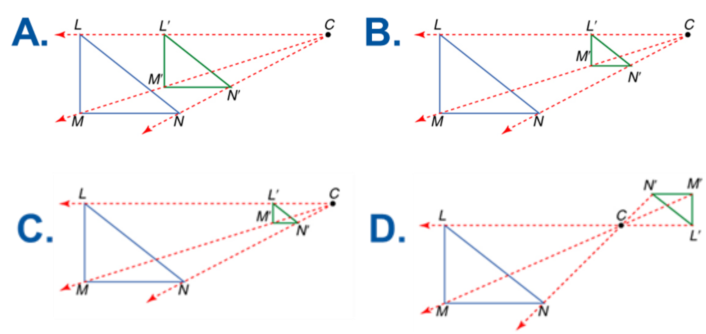
$\triangle L'M'P'$ is the image of $\triangle LMP$ under a dilation with center C and scale factor 2.5.

Example 7-6-6: **Draw a Dilation**

A. Copy trapezoid *PQRS* and point *C*. Then use a ruler to draw the image of trapezoid *PQRS* under a dilation with center *C* and scale factor 3.



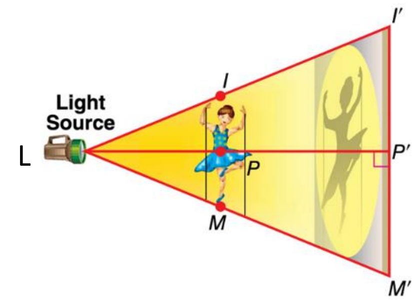
B. Which diagram shows the dilation image of ΔLMN with center C and $k = \frac{1}{3}$?



Example 7-6-7: **Find the scale factor of a Dilation**

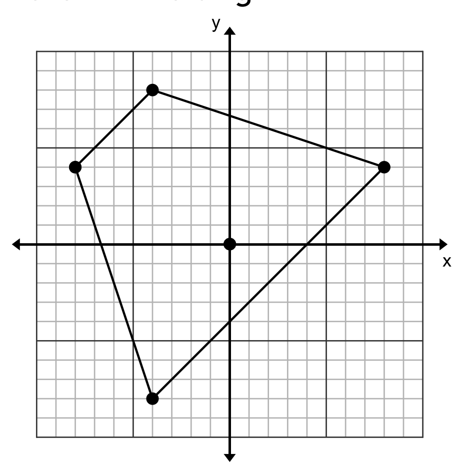
To create the illusion of a "life-sized" image, puppeteers sometimes use a light source to show an enlarged image of a puppet projected on a screen or wall. Suppose that the distance between a light source *L* and the puppet is 24 inches (*LP*). To what distance *PP′* should you place the puppet from the screen to create a 49.5-inch tall shadow (*I′M′*) from a 9-inch puppet?

$\frac{puppet}{shadow} =$



Example 7-6-8: **Dilations in the coordinate plane.**

A. Trapezoid *EFGH* has vertices *E*(–8, 4), *F*(–4, 8), *G*(8, 4) and *H*(–4, –8). Graph the image of *EFGH* after a dilation centered at the origin with a scale factor of $\frac{1}{4}$



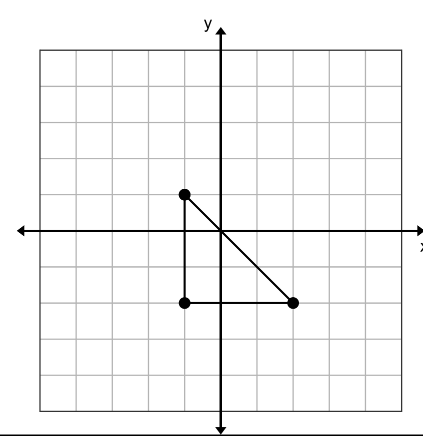
$\frac{1}{4}E \rightarrow E'(\quad , \quad)$

$\frac{1}{4}F \rightarrow F'(\quad , \quad)$

$\frac{1}{4}G \rightarrow G'(\quad , \quad)$

$\frac{1}{4}H \rightarrow H'(\quad , \quad)$

B. Triangle *ABC* has vertices *A*(–1, 1), *B*(2, –2), and *C*(–1, –2). Graph the image of Δ*ABC* after a dilation centered at the origin with a scale factor of 2.



$2A \rightarrow A'(\quad , \quad)$

$2B \rightarrow B'(\quad , \quad)$

$2C \rightarrow C'(\quad , \quad)$