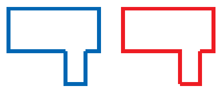
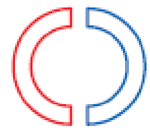
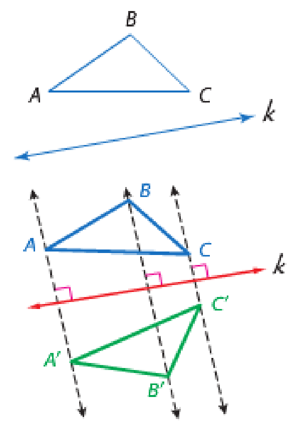
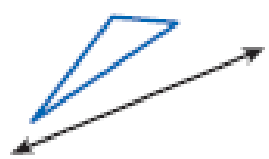
Geometry Unit 2 – Foundations of Geometry
Chapter 9

Friday September 13	9-1 Reflections
Monday September 16	9-2 Translations DHQ 9-1
Tuesday September 17	9-3 Rotations DHQ 9-2
Block Wed/Thurs. Sept 18/19	Review 9-1 to 9-3 Quiz 9-1 and 9-2 9-4 Composition DHQ 9-3
Friday September 20	9-5 Symmetry DHQ 9-4
Monday September 23	Review Transformations
Tuesday September 24	Review Transformations
Block Wed/Thurs. Sept 25/26	Test Transformations
Friday September 27	No School – Teacher Work Day
Monday September 30	Start Unit 3 – Chapter 2 – Reasoning and Proof

"It is not enough to have a good mind. The main thing is to use it well."

— Reneé Descartes

Reflections – 9.1

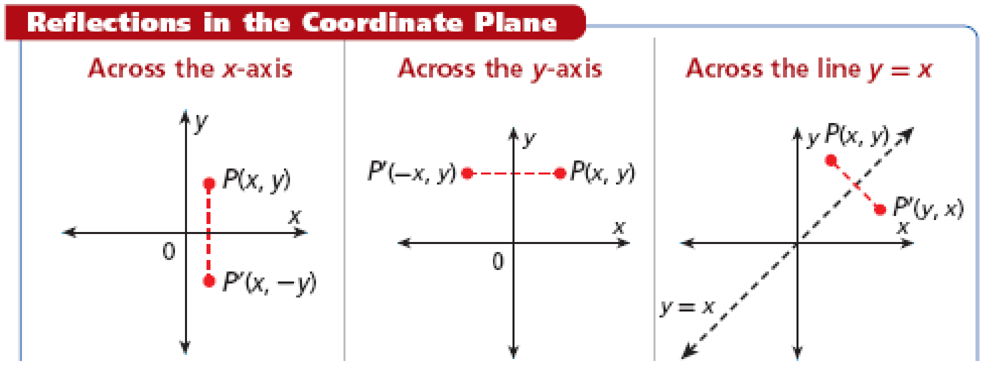
If two figures are congruent, is one necessarily a reflection of the other? How do you know when a transformation is a reflection?	An *isometry* can also be referred to as a ____________ transformation. A ___________________ is a transformation with rigid motion where every point is the ___________ ____________________ from the line of reflections. Example 1: Identifying Reflections Tell whether each transformation appears to be a reflection. A.  B. 
	 **Example 1** Reflect a Figure in a Line **Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.** **Step 1** Draw a line through each vertex that is perpendicular to line k. **Step 2** Measure the distance from point A to line k. Then locate A' the same distance from line k on the opposite side **Step 3** Repeat Step 2 to locate points B' and C'. Then connect vertices A', B', and C' to form the reflected image.
When given the preimage and the image, how could you locate the line of reflection?	Example 2 – Drawing Reflections Copy the triangle and the line of reflection. Draw the reflection of the triangle across the line. 

$\triangle JKQ \rightarrow \triangle J'K'Q'$

How we say it:

$\triangle JKQ$ maps onto $\triangle J'K'Q'$.

Notation: A transformation maps every point of a figure onto its image and may be desribed with arrow notation → Prime notation (') is sometimes used to identify image points.

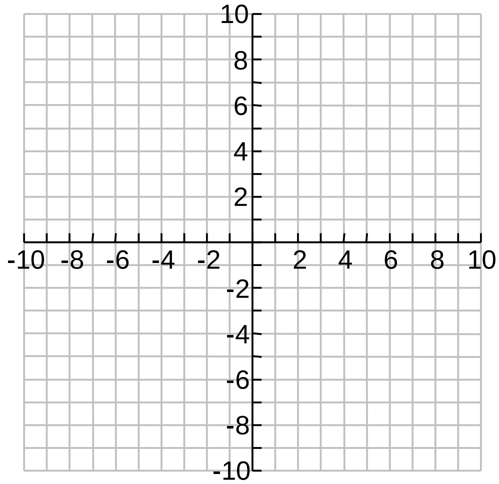


Type of Reflection	Rule (mapping)
Reflection in the x−axis	$(x, y) \rightarrow$
Reflection in the y−axis	$(x, y) \rightarrow$
Reflection in the line $y = x$	$(x, y) \rightarrow$

Example 3 – Drawing Reflections in the Coordinate Plane

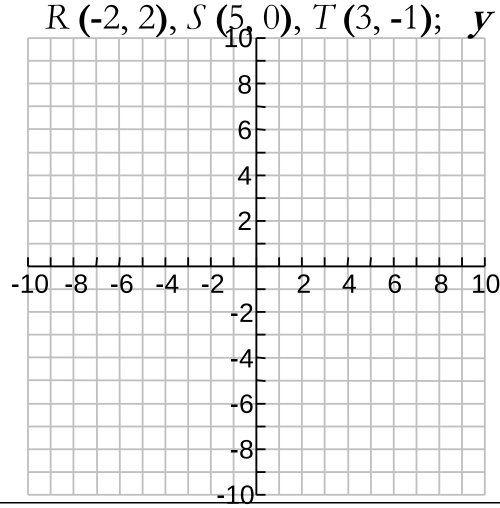
Reflect the figure with the given vertices across the given line.

A. X **(**2, **-**1**)**, Y **(-**4, **-**3**)**, Z **(**3, 2**)**; ***x*-axis** **Rule:** ____________



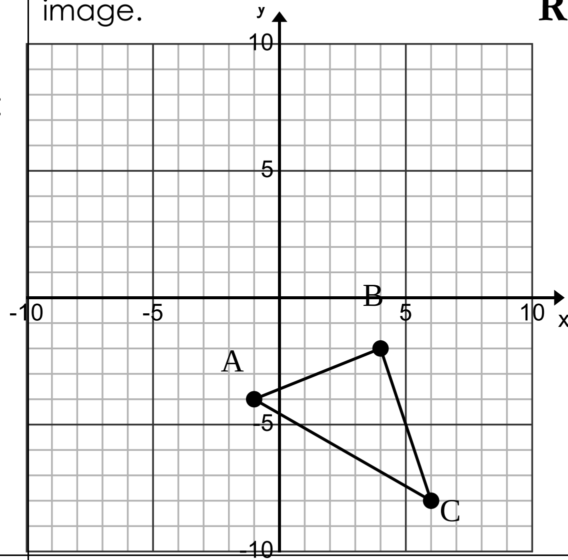
X’:
Y’:
Z’:

B) R **(-**2, 2**)**, S **(**5, 0**)**, T **(**3, **-**1**)**; ***y* = *x*** **Rule:** ____________



R’:
S’:
T’:

How is reflecting across the y-axis different than reflecting across the x-axis?	C. Reflect the pre-image across the **y-axis.** Give the coordinates of the final image. **Rule:** ____________



A: (-1, -4)	A':
B: (4, -2)	B':
C: (6, -8)	C':

REVIEW

Reflection in the *x*-axis		Reflection in the *y*-axis	
Words	To reflect a point in the *x*-axis, multiply its *y*-coordinate by −1.	Words	To reflect a point in the *y*-axis, multiply its *x*-coordinate by −1.
Symbols	$(x, y) \rightarrow (x, -y)$	Symbols	$(x, y) \rightarrow (-x, y)$

Reflection in Line $y = x$	
Words	To reflect a point in the line $y = x$, interchange the *x*- and *y*-coordinates.
Symbols	$(x, y) \rightarrow (y, x)$

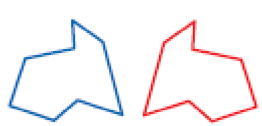
Translations – 9.2

What is the difference between a translation and a reflection?	A translation is a transformation where all the points of a figure are moved the same distance in the same direction.

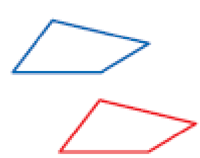
Example 1 – Identifying Translations

Tell whether each transformation appears to be a translation.

A.



B.



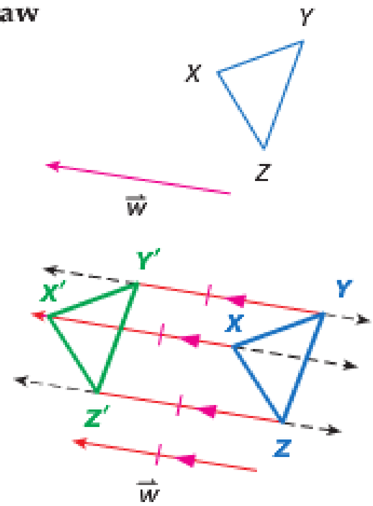
Example 1 Draw a Translation

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

Step 1 Draw a line through each vertex parallel to vector $\vec{w}$.

Step 2 Measure the length of vector $\vec{w}$. Locate point X' by marking off this distance along the line through vertex X, starting at X and in the same direction as the vector.

Step 3 Repeat Step 2 to locate points Y' and Z'. Then connect vertices X', Y', and Z' to form the translated image.

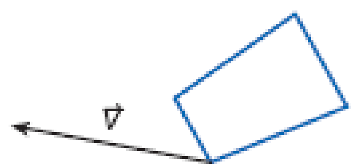


How do you know that the distance between each point and its image is the same for each pair of corresponding points?

How do you know that the direction each point has moved is the same for each pair of corresponding points?

Example 2 – Drawing Translations

Copy the quadrilateral and the translation vector. Draw the translation along the vector.



Vectors 8.7

Identify the component form of a vector.

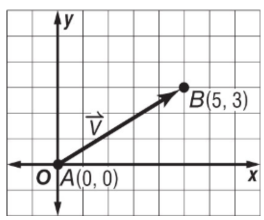
A translation is a transformation where all the points of a figure are moved the same distance in the same direction.

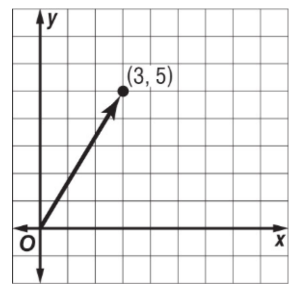
The method that it is moved by is through the use of a **vector**. In mathematics a vector is a way to identify the direction that one point is moved in the coordinate plane. Vector $\boldsymbol{v}$ is identified in one of two ways:

$$\vec{\boldsymbol{v}} = \langle a, b \rangle \qquad \text{or} \qquad \vec{\boldsymbol{v}} = (x + a, y + b)$$

Where a and b indicate how far a point is moved in the x and y direction, respectively.

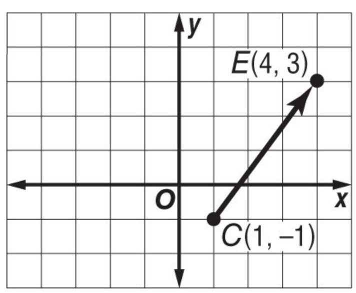
What is a vector? It basically just tells us how far to move each point in the x and y directions.

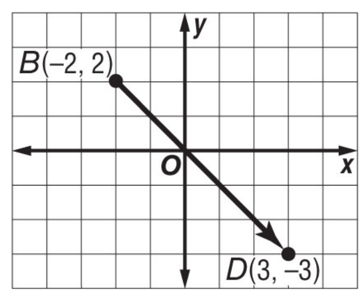




Component form: _________ Component Form: ________

What if it’s doesn’t start at the origin?



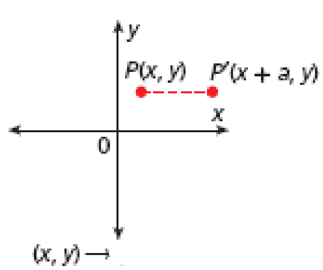


Component form: _________ Component Form: ________

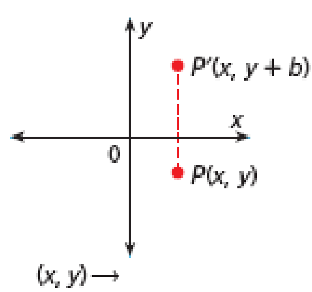
So how do we apply this to translations? Let’s see…

Translations in the Coordinate Plane

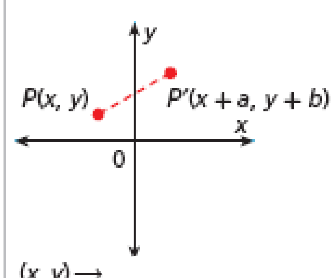
Horizontal Translation Along Vector ⟨a, 0⟩

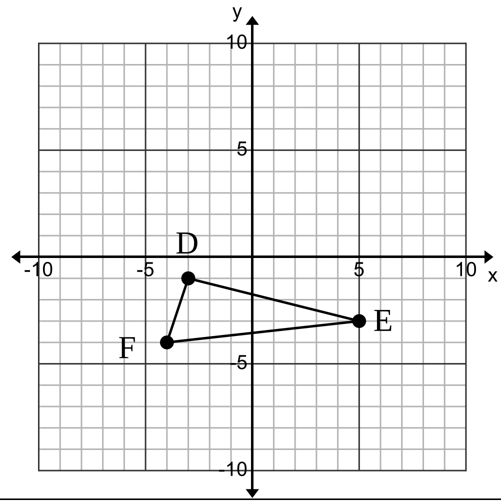
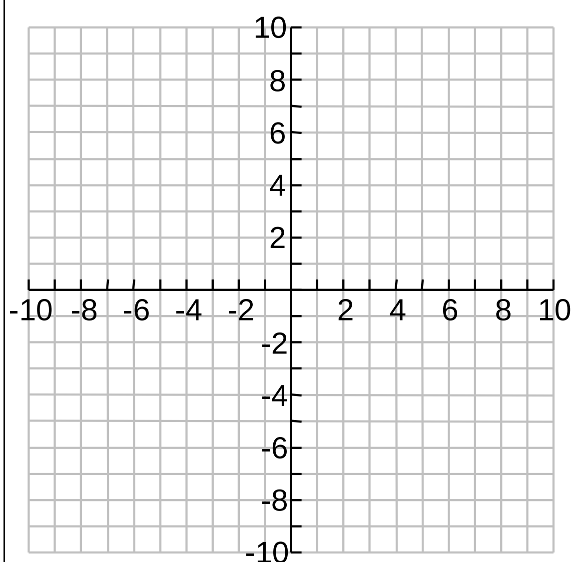


Vertical Translation Along Vector ⟨0, b⟩



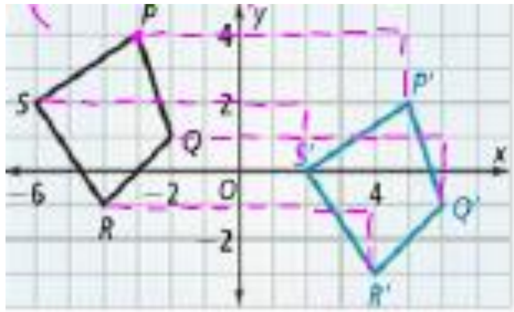
General Translation Along Vector ⟨a, b⟩



How is the translation vector related to $\overline{DD'}$?	Example 3 – Drawing Translations in the Coordinate Plane Translate the triangle with vertices $D(-3,-1)$, $E(5,-3)$, $F(-4,-4)$ along the vector $\langle -4, 7\rangle$  D’ (,) E’(,) F’(,) Equation: (x,y) → (,)
How do you find the vector that moves an object directly from its starting position to its final position?	Example 4 – Real World Application  Mrs. Rushing is at the point (2, 7). Mrs. McWhorter is at the point (-1, -5). Ms. Waldron is at point (6, 2). Mrs. Bajich is at point (-7, 2). When Mrs. Rushing throws the frisbee to Mrs. Bajich, what vector path does it follow? If Mrs. Rushing throws to Mrs. Bajich, who throws to Ms. Walron, who throws to Mrs. McWhorter, then what vector would Mrs. McWhorter have to throw to reach Mrs. Rushing?

Example 5

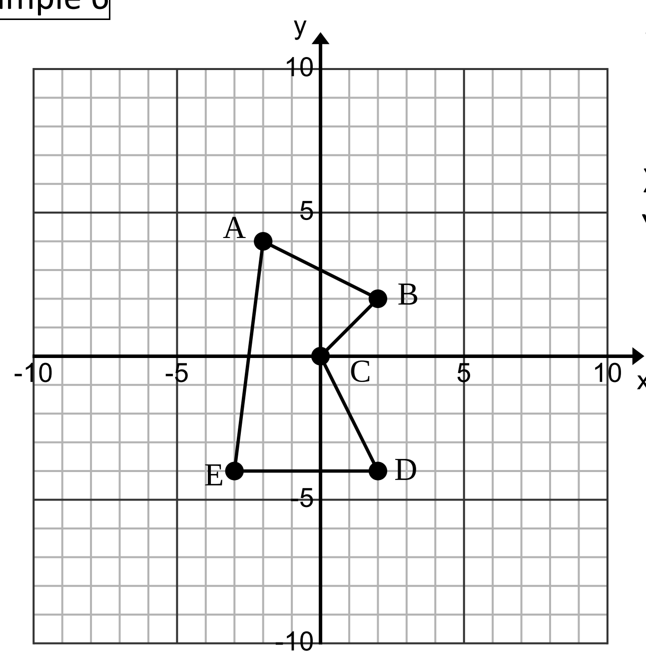
What is the rule that describes the translation that maps PQRS onto P'Q'R'S'?



Create an **equation** in terms of x and y for this translation.

What is the vector? __________

Example 6



TranslationT <3, -2> (ABCDE)

X moves _________ 3

Y moves __________ 2

A (____, _____)
B (____, _____)
C (____, _____)
D (____, _____)
E (____, _____)

A' (____, _____)
B' (____, _____)
C' (____, _____)
D' (____, _____)
E' (____, _____)

Rotations – 9.3

A _________________ is a transformation in which a figure is _________ about

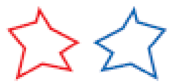
a ___________ point. The ___________ point is the __________ ___ __________.

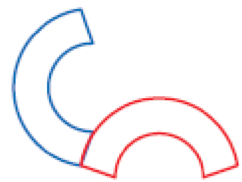
Rotations can be clockwise or counterclockwise.

How is a rotation different from a reflection?

Example 1 – Identifying Rotations

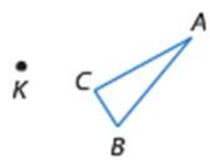
Tell whether each transformation appears to be a rotation.

A. 

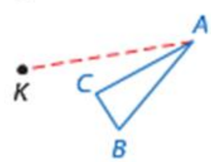
B. 

Example 1 Draw a Rotation

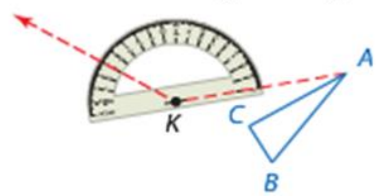
Copy $\triangle ABC$ and point K. Then use a protractor and ruler to draw a 140° rotation of $\triangle ABC$ about point K.



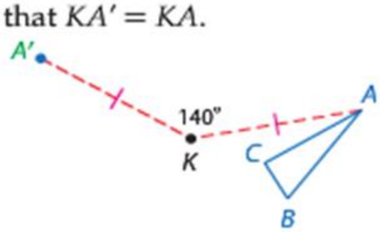
Step 1 Draw a segment from A to K.



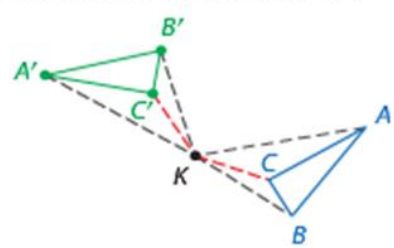
Step 2 Draw a 140° angle using $\overline{KA}$.



Step 3 Use a ruler to draw A' such that $KA' = KA$.



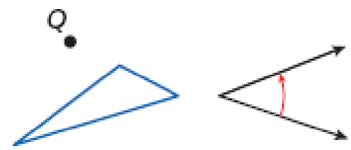
Step 4 Repeat Steps 1–3 for vertices B and C and draw $\triangle A'B'C'$.

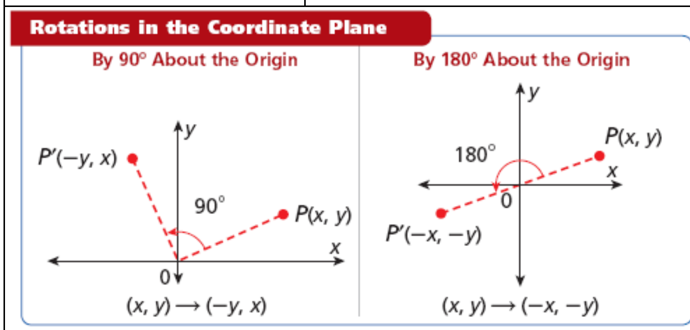


How can you verify your answer?

Example 2 – Drawing Rotations

Copy the figure and the angle of rotation. Draw the rotation of the triangle about point *Q* by $\angle A$.





Type of Translation	Rule (mapping)
90° about the origin	$(x, y) \rightarrow$
180° about the origin	$(x, y) \rightarrow$
270° about the origin	$(x, y) \rightarrow$

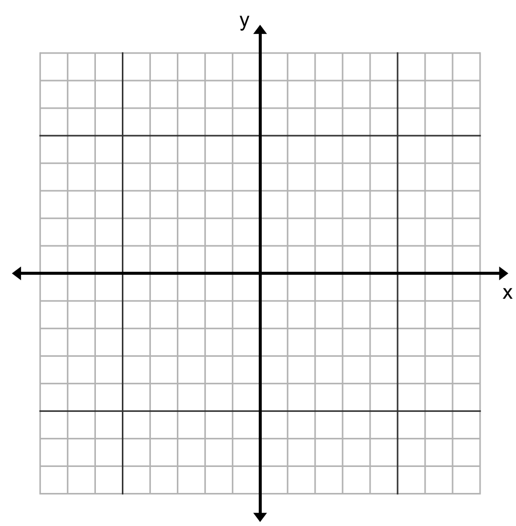
If all three vertices of a triangle are in Quadrant II and the triangle is rotated 90° about the origin, in which quadrant will the vertices of the rotated triangle lie?

Example 3 – Drawing Rotations in the Coordinate Plane

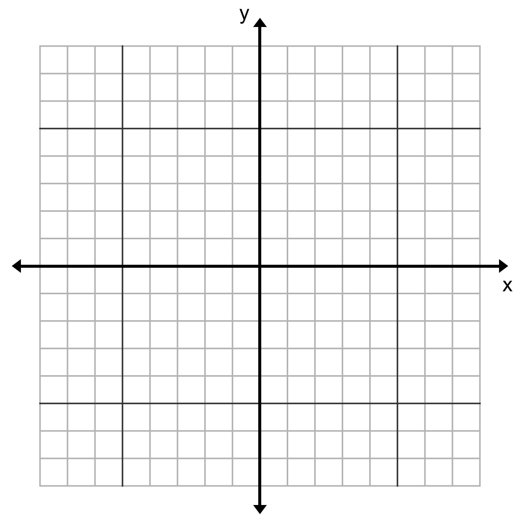
A. Rotate ΔJKL with vertices *J* **(**2, 2**)**, *K* **(**4, –5**)**, and *L* **(**–1, 6**)** by **180°** about the origin.

Rule: __________

J′ **(** , **)** *K′* **(** , **)** *L′* **(** , **)**



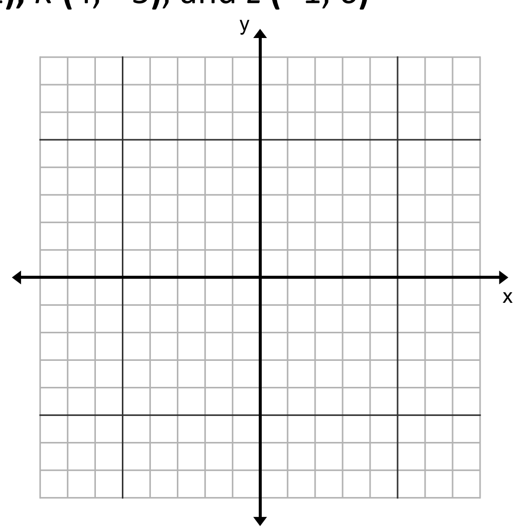
B. Rotate ΔJKL with vertices *J* **(**2, 2**),** *K* **(**4, −5**)**, and *L* **(**−1, 6**)** by **90°** about the origin.



Rule: _________

J' **(** , **)** *K'* **(** , **)** *L'* **(** , **)**

C. Rotate ΔJKL with vertices *J* **(**2, 2**),** *K* **(**4, −5**)**, and *L* **(**−1, 6**)** by **270°** about the origin.

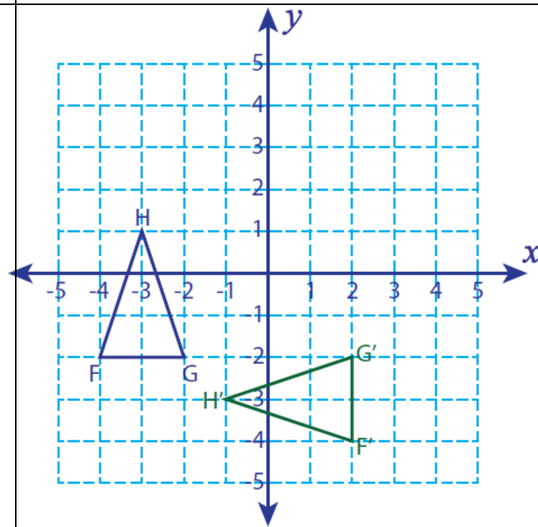
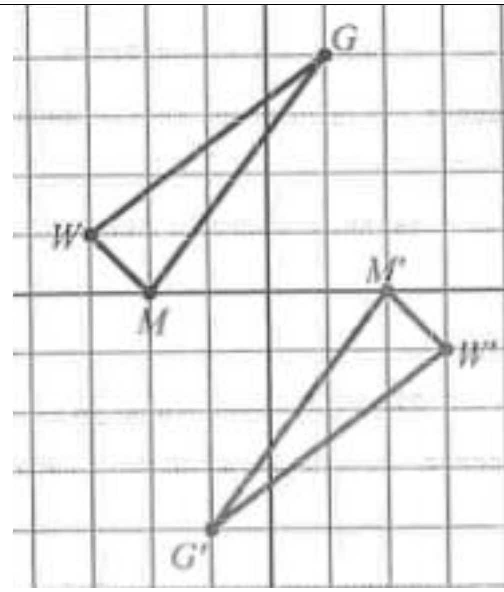
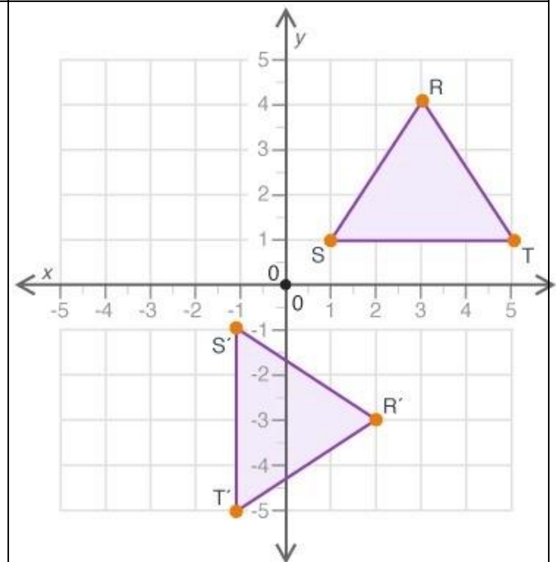


Rule: _________

J' **(** , **)** *K'* **(** , **)** *L'* **(** , **)**

Rotation Exploration

Directions: What pattern do you notice happens each time between the coordinates of the preimage and the coordinates of the image?

	Rotation= 90° **counterclockwise** about the origin	Rotation= 180° **counterclockwise** about the origin	Rotation= 270° **counterclockwise** about the origin
			
Ordered pairs of preimage	F (-4, -2) H (-3, 1) G (-2, -2)	W (-3, 1) G (1, 4) M (-2, 0)	S (1, 1) R (3, 4) T (5, 1)
Ordered pairs of image	F' (2, -4) H' (-1, -3) G' (2, -2)	W' (3, -1) G' (-1, -4) M'(2, 0)	S' (-1, -1) R' (2, -3) T' (-1, -5)
Observation/ Rule			

Composition of Transformation – 9.4

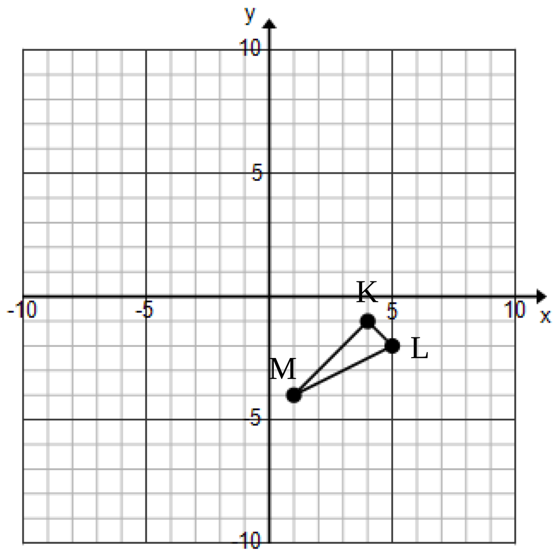
Composition of Transformations -> a combination of two or more transformations, each performed on the previous image.

Glide Reflection -> a combination of a **reflection** and a translation.

A composition of two isometries is an isometry.

Example 1 – Drawing Compositions of Isometries

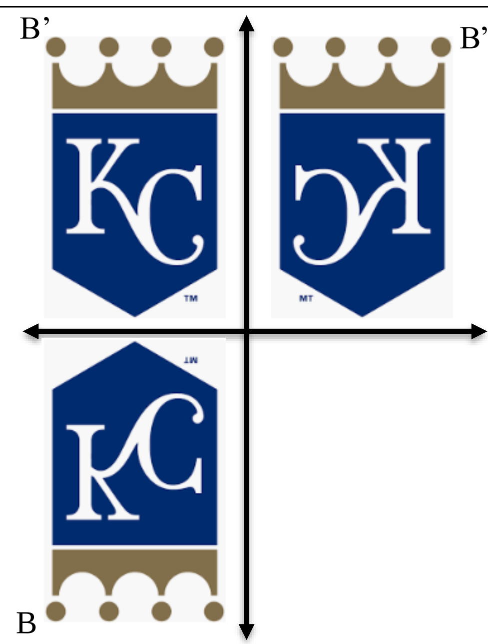
ΔKLM has vertices K (4, -1), L (5, -2), and M (1, -4). Rotate ΔKLM 180° about the origin and then reflect it across the y-axis.



Rotate	Reflect
K’	K’’
L’	L’’
M’	M’’

Example 2

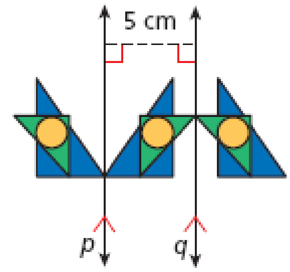
Take the logo for the royals shown… If you were to describe the motion from B to B’ to B’’ with only a single transformation, what would you call it?



Is the answer the same if the order of transformations is reversed?

Example 3 – Real World Application

Sean reflects a design across line *p* and then reflects the image across line *q*. Describe a single transformation that moves the design from the original position to the final position.

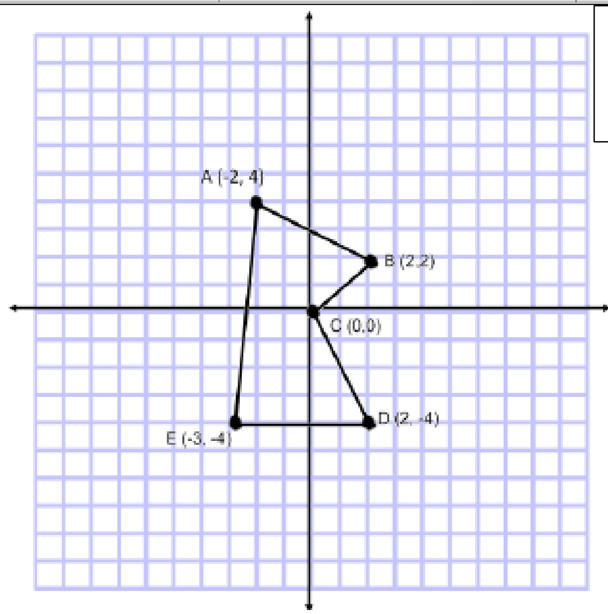


Any translation or rotation is equivalent to a composition of ________ reflections.

ConceptSummary Compositions of Translations

Glide Reflection	Translation	Rotation
the composition of a reflection and a translation	the composition of two reflections in parallel lines	the composition of two reflections in intersecting lines

How is the orientation of an image related to its preimage after a glide reflection?



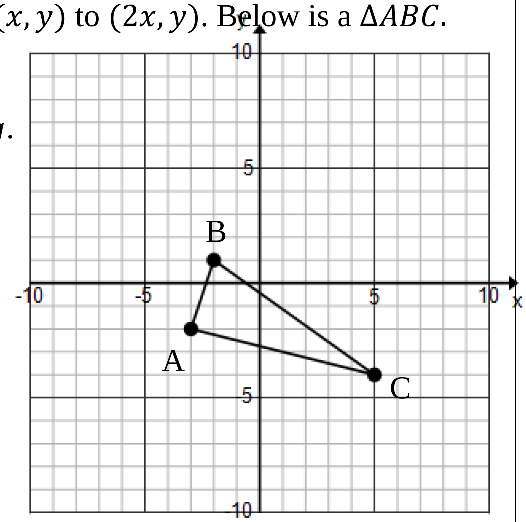
Reflect across the y-axis
AND
Translate $< 3, -6 >$

A' (___, ___)
B' (___, ___)
C' (___, ___)
D' (___, ___)
E' (___, ___)

A'' (___, ___)
B'' (___, ___)
C'' (___, ___)
D'' (___, ___)
E'' (___, ___)

Example 4 Suppose g is the map of the plane that takes each (x, y) to $(2x, y)$. Below is a ΔABC.

$A(-3, -2)$ $B(-2, 1)$ $C(5, -4)$

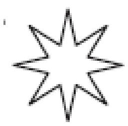
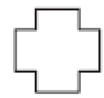
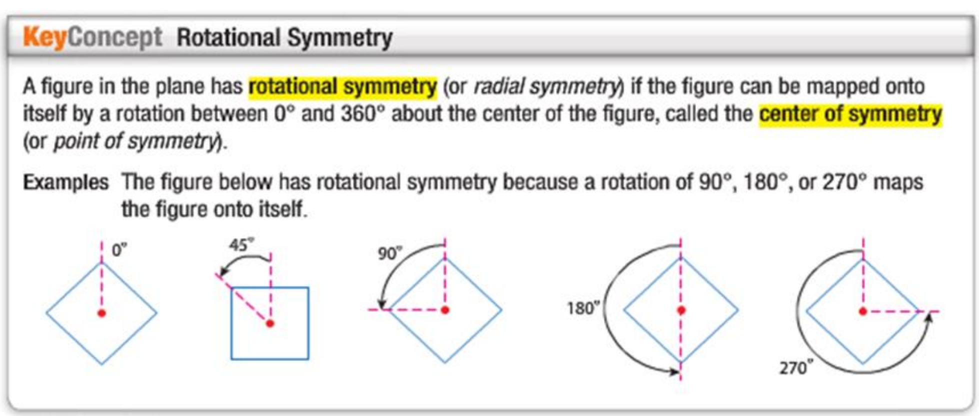


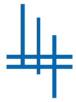
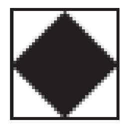
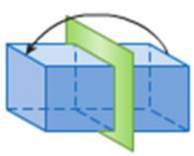
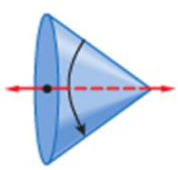
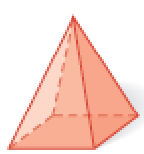
a) Show the image of △ABC after applying the map of g.

A' (____,____)
B' (____,____)
C' (____,____)

b) Does g preserve distances and angles? Explain.

Symmetry – 9.5

	Symmetry -> When two or more parts are identical after a translation, reflection or rotation. **Line of Symmetry**: A line that divides a figure into two parts such that, when the figure is folded along the line, the two parts of the figure coincide. 
How do you locate a figures line of symmetry?	Example 1 – Identifying Line Symmetry Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry. A.  Yes No B.  Yes No C.  Yes No
	Example 2 Draw a figure with 3 lines of symmetry:
	A figure has **rotational symmetry** (or *radial symmetry*) if it can be **rotated** about a point by an angle greater than 0° and less than 360° so that the image coincides with the pre-image. **Order of symmetry** – the number of times the figure maps onto itself as it rotates **Magnitude of symmetry** – angle of rotation (Magnitude $=$ $360 \div$ order)  **KeyConcept Rotational Symmetry** A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure, called the **center of symmetry** (or *point of symmetry*). **Examples** The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.

How do you determine a figure's angle of rotational symmetry?	Example 3 – Identifying Rotational Symmetry Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order. A.  Yes No Angle rotation: ______ Order: ____ B.  Yes No Angle rotation: ______ Order: ____ C.  Yes No Angle rotation: ______ Order: ____
	Example 4 – Real World Application Describe the symmetry of each icon. Copy each shape and draw any lines of symmetry. If there is rotational symmetry, give the angle and the order. A.  Line or Rotational Symmetry # of lines: ______ Angle rotation: ______ Order: _____ B.  Line or Rotational Symmetry # of lines: ______ Angle rotation: ______ Order: _____
 	A three-dimensional figure has ***plane symmetry*** if a plane can divide the figure into two congruent reflected halves. A three-dimensional figure has ***symmetry about an axis*** if there is a line about which the figure can be rotated (by an angle greater than 0° and less than 360°) so that the image coincides with the pre-image.
How can you tell if a 3-d figure has plane symmetry?	Example 5 – Identifying Symmetry in Three Dimensions Tell whether each figure has plane symmetry, symmetry about an axis, or neither. A. square pyramid  B.